

# Spurious correlation of $I(1)$ regressors in models with an $I(0)$ dependent variable: simulation results

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## Abstract

Using asymptotic theory Stewart (2007) demonstrated that the t-ratios (F-statistic) on  $I(1)$  regressors in a model with an  $I(0)$  dependent variable (with or without additional  $I(0)$  regressors) converge to a random variable and not zero. This paper provides simulation results which indicate that these t-statistics will, in general, exhibit spurious correlation. This suggests that spurious regression is more widespread than previously thought. Further, because the specification considered is of the error-correction form (under the null of no cointegration) this spurious correlation is argued to potentially give rise to spurious cointegration. No other statistics in this specification exhibit spurious correlation.

Key words: spurious correlation, Monte Carlo simulation, unbalanced regression, spurious cointegration.

JEL classification: C01, C15

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## 1. Introduction

Spurious regression refers to exaggerated correlation indicated by various statistics in a (typically) linear regression model (LRM). This could be manifested in exaggerated magnitudes of coefficients, overstated absolute values of t-ratios, an inflated  $R^2$ , a too low regression standard error and/or excessively large F-statistics. The focus has typically been on the appearance of correlation between independently generated variables as indicated by t- and F-tests for zero restrictions on coefficients.<sup>2</sup> In particular, spurious regression occurs when two or more independently generated variables are regressed upon each other and the coefficients of the regressors are found to be statistically significant more frequently than the specified nominal level of significance. This will be the case if t- or F-test statistics for zero restrictions diverge to infinity asymptotically, because they will generally exceed (in magnitude) their corresponding critical values.

Granger and Newbold (1974), Phillips (1986) and Entorf (1997) demonstrate that there will be spurious regression on the slope coefficients involving Ordinary Least Squares (OLS) regressions of independently generated  $I(1)$  variables. Spurious correlations are also shown to be found in regressions involving random walks and linear trends according to Nelson and Kang (1984) and Durlauf and Phillips (1988). Haldrup (1994) and Marmol (1995 and 1996) demonstrate that spurious correlations are evident in OLS regressions involving combinations of series with integer orders of integration equal to, or greater than, one. Spurious regressions are shown to occur in models with series generated by various combinations of different types of stationary process (with and without linear trends and possibly allowing for time-varying means due to structural breaks or seasonality) by Jenkins and Watts (1968), Granger *et al* (2001), Hassler (2003) and Kim *et al* (2004). Marmol (1998), Tsay and Chung (2000) and Sun (2006) demonstrate that spurious correlation generally occurs in regressions involving various combinations of fractionally integrated processes (both stationary and nonstationary).<sup>3</sup>

Hassler (1996) derived the asymptotic theory for regressions of an  $I(1)$  dependent variable on a set of  $I(0)$  regressors [both including and excluding further  $I(1)$  explanatory factors], where  $\hat{\beta}_1$  is the vector of coefficients on the  $I(0)$  terms in the LRM. For this case it is noted that, "... it is somehow surprising that  $\hat{\beta}_1$  does not converge to zero. Nevertheless, we observe no spurious significance of  $\hat{\beta}_1$ ... meaning that t- and F-statistics do not diverge." (Hassler, 1996, p. 29). However, Stewart (2006) argues that non-divergent t-ratios and F-tests does not automatically imply the absence of spurious regression. For example, the empirical size of a t-ratio that converges to a random variable can still exceed its nominal size if the mean of that random variable is large relative to its critical value. Simulation evidence was presented to support this claim.

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<sup>2</sup> From the point of view of the applied econometrician it is t- and F-tests that are used to determine whether variables are significantly correlated with the dependent variable and thus indicate whether they should remain in the model. Hence, the emphasis is on such t- and F-statistics, rather than particular coefficient values or even the  $R^2$ , in terms of whether erroneous inference is being drawn.

<sup>3</sup> There is no spurious regression in some cases involving stationary series. Sun (2006) considers generalised fractionally integrated (Gegenbauer) processes.

Stewart (2007) has derived the asymptotic theory for the reverse of this case: the regression of an  $I(0)$  dependent variable on a set of  $I(1)$  regressors [both including and excluding further  $I(0)$  explanatory factors]. Amongst other things it is found that, when the  $I(0)$  terms follow stationary first-order autoregressive processes, denoted AR(1), the t-ratio on the  $I(1)$  regressor asymptotically converges to a random variable. This paper uses simulation evidence to determine whether spurious correlation can arise in this case.

The next section discusses and summarises the asymptotic results for this type of model. Simulation results are provided in section 3. Section 4 summarises and concludes.

## 2. Summary of Asymptotic Theory

Stewart (2007) derives the asymptotic theory for model (1), and the unbalanced regression, which is equation (1) with  $\hat{\beta}_1 = 0$ .

$$Y_{0t} = \hat{\alpha} + \hat{\beta}_1' \underline{X}_{1t} + \hat{\beta}_2' \underline{X}_{2t} + \hat{u}_t \quad (1)$$

where,  $Y_{0t}$  is the stationary AR(1) dependent variable,  $\underline{X}_{1t}$  is an  $m_1 \times 1$  vector of stationary AR(1) processes,  $\underline{X}_{2t}$  is an  $m_2 \times 1$  vector of  $I(1)$  random walks (without drifts) and  $\hat{\beta}_1$  and  $\hat{\beta}_2$  are the respective  $m_1 \times 1$  and  $m_2 \times 1$  coefficient vectors on the regressors. The intercept is  $\hat{\alpha}$  and  $\hat{u}_t$  is the residual. All variables are assumed to be independently generated.

Stewart (2007) demonstrates that  $\hat{\alpha} = o_p(T^{-0.5})$ ,  $\hat{\beta}_1 = o_p(T^{-0.5})$ ,  $\hat{\beta}_2 = o_p(T^{-1})$ ,  $t_{\hat{\beta}_1} = O_p(1)$ ,  $t_{\hat{\beta}_2} = O_p(1)$ ,  $F_{\hat{\beta}_1} = O_p(1)$ ,  $F_{\hat{\beta}_2} = O_p(1)$ ,  $s^2 = O_p(1)$ ,  $R^2 = o_p(T^{-1})$  and  $d = O_p(1)$  for model (1). Where  $t_{\hat{\beta}_1}$  and  $t_{\hat{\beta}_2}$  denote the t-statistics for testing that the individual coefficients on the  $I(0)$  and  $I(1)$  variables, respectively, are zero.  $F_{\hat{\beta}_1}$  and  $F_{\hat{\beta}_2}$  are respective F-tests of zero restrictions on all of the  $I(0)$  variables' coefficients and all of the  $I(1)$  variables' coefficients. Further,  $s^2$  is the residual variance estimator,  $R^2$  is the coefficient of determination and  $d$  is the Durbin-Watson statistic.<sup>4</sup> Therefore,  $\hat{\alpha}$ ,  $\hat{\beta}_1$ ,  $\hat{\beta}_2$  and  $R^2$  all converge to zero asymptotically while

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<sup>4</sup> The notation  $O_p(T^\delta)$  means that a statistic divided by  $T^\delta$  converges in the probability limit to a (finite non-zero) random variable (typically expressed as a function of Brownian motions). Equivalently, the unscaled statistic would be said to asymptotically diverge to infinity at rate  $T^\delta$ . The notation  $o_p(T^\delta)$  means that a statistic divided by  $T^\delta$  converges in the probability limit to zero – the numerator is of a smaller order in probability than the denominator.

$t_{\hat{\beta}_1}$ ,  $t_{\hat{\beta}_2}$ ,  $F_{\hat{\beta}_1}$ ,  $F_{\hat{\beta}_2}$ ,  $s^2$  and  $d$  all converge to random variables in the probability limit.<sup>5</sup>

The unbalanced regression, equation (1) with  $\hat{\beta}_1 = 0$ , is:

$$Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2' X_{2t} + \tilde{u}_t \quad (2)$$

Stewart (2007) demonstrates that  $\tilde{\alpha} = o_p(T^{-0.5})$ ,  $\tilde{\beta}_2 = o_p(T^{-1})$ ,  $t_{\tilde{\beta}_2} = O_p(1)$ ,  $F_{\tilde{\beta}_2} = O_p(1)$ ,  $\tilde{s}^2 = O_p(1)$ ,  $\tilde{R}^2 = o_p(T^{-1})$  and  $\tilde{d} = O_p(1)$  for model (2).  $t_{\tilde{\beta}_2}$  denotes the t-statistics for testing that the individual coefficients on the  $I(1)$  variables are zero.  $F_{\tilde{\beta}_2}$  is the F-test for zero restrictions on all of the  $I(1)$  variables' coefficients. Similar to model (1),  $\tilde{s}^2$  is the residual variance estimator,  $\tilde{R}^2$  is the coefficient of determination and  $\tilde{d}$  is the Durbin-Watson statistic. This means that,  $\tilde{\alpha}$ ,  $\tilde{\beta}_2$  and  $\tilde{R}^2$  all converge to zero asymptotically while  $t_{\tilde{\beta}_2}$ ,  $F_{\tilde{\beta}_2}$ ,  $\tilde{s}^2$  and  $\tilde{d}$  all converge to random variables in the probability limit.

For both model (1) and (2) it is expected that all of the coefficients and the coefficient of determination should converge to zero because all the variables in the regression are independently generated zero mean processes. The asymptotic theory cited above unambiguously confirms that these statistics converge to their expected values of zero. Hence these statistics do not exhibit spurious correlation in large samples.

In contrast, it is unclear whether the t-ratios, F-statistics and the estimated regression error variance feature spurious regression because they all converge to random variables. For the t-ratios and F-tests to be subject to spurious correlation the empirical size of the test would need to significantly exceed the nominal size of the test.<sup>6</sup> Further, if  $Y_{0t}$  is a stationary, zero mean AR(1) process with unit variance the regression error variance should converge to the following asymptotically (see the appendix):<sup>7</sup>

$$Var(u_t) = \left( \frac{1}{1 - \rho^2} \right) \quad -1 < \rho < 1 \quad (3)$$

<sup>5</sup> Some of the latter set of statistics converge to constants.

<sup>6</sup> Granger *et al* (2001, Table 2, p. 901) report simulation results for the slope coefficient's t-ratio,  $t_{\tilde{\beta}_2}$ , in the unbalanced regression [model (2)] when the single regressor is AR(1), with autocorrelation coefficient equal to 0.5. The reported rejection probabilities (with a nominal size of 5%) are 26.4%, 26.4% and 24.3% for sample sizes of 100, 500 and 2000, respectively, indicating spurious regression. Granger *et al* (2001) do not comment upon these results: they appear to be an externality in their analysis of spurious correlation among stationary variables.

<sup>7</sup> This is the population variance for both model (1) and model (2), given the latter is encompassed by the former.

Where  $\rho$  is the autocorrelation coefficient in the data generation process (DGP) of  $Y_{0t}$  - see equation (5) below.<sup>8</sup>

If the estimated residual variance converges to a value below that predicted by (3) this statistic features evident spurious regression.

For both model (1) and (2) Stewart (2007) shows that the Durbin-Watson statistic converges to  $2(1 - \hat{\rho})$  in the probability limit, where  $\hat{\rho}$  is the OLS estimator of  $\rho$ . Because all coefficients have been shown to converge to zero the regression error term [of both (1) and (2)] should feature exactly the same AR(1) process as the dependent variable. Thus, the OLS estimator,  $\hat{\rho}$ , will be consistent if  $|\rho| < 1$  which implies that:

$$p\lim d = 2(1 - \rho) \quad \text{and} \quad p\lim \tilde{d} = 2(1 - \rho) \quad (4)$$

Therefore, as the degree of autocorrelation in  $Y_{0t}$  rises the Durbin-Watson statistic will increasingly indicate residual autocorrelation. Further, the exact value that the DW statistic should converge to is given by (4).

The next section will discuss the Monte Carlo simulation experiments for model (1) and (2). The aim will be to verify that the coefficients and coefficient of determination do not exhibit spurious regression, to determine whether the t-ratios and regression standard error are indicative of spurious correlation and to assess whether the predicted value of the Durbin-Watson statistic is achieved.<sup>9</sup>

### 3. Monte Carlo Simulation Experiments

For the Monte Carlo simulation experiment we specify the DGPs of the three variables,  $X_{1t}$ ,  $X_{2t}$  and  $Y_{0t}$ , as:

$$Y_{0t} = \rho Y_{0,t-1} + e_{0t} \quad Y_0 = 0 \quad e_{0t} \sim N(0, 1) \quad (5)$$

$$X_{1t} = \phi X_{1,t-1} + e_{1t} \quad X_{1,0} = 0 \quad e_{1t} \sim N(0, 1) \quad (6)$$

$$X_{2t} = X_{2,t-1} + v_{2t} \quad X_{2,0} = 0 \quad v_{2t} \sim N(0, 1) \quad (7)$$

where  $X_{1,0}$ ,  $X_{2,0}$  and  $Y_0$  denote the initial values of the variables (all are set to zero),  $\rho$  and  $\phi$  are autocorrelation coefficients and  $e_{0t}$ ,  $e_{1t}$  and  $v_{2t}$  are stochastic errors generated using independent standard normal distributions.

The model to be estimated is:

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<sup>8</sup> If  $\rho = 1$  the expected population variance, given a unit error variance of the dependent variable's DGP, is  $Var(u_t) = T$ .

<sup>9</sup> We do not consider F-tests because they will yield exactly the same results as those produced with t-tests given the models that we employ for simulation. That is, the models considered include, at most, one  $I(1)$  regressor and one  $I(0)$  regressor.

$$Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t \quad (8)$$

Equation (8) is estimated by OLS for various sample sizes ( $T = 25, 50, 100, 200, 500, 1000, 5000, 10000$ ),<sup>10</sup> and values of the autocorrelation coefficients,  $\rho$  and  $\phi$ , ( $0 \leq \rho \leq 1$  and  $0 \leq \phi \leq 1$ , rising in 0.1 unit increments). From the 10,000 replications we calculate the following for model (1) and (2):

- (a) The average magnitudes of the OLS coefficient estimators (the averages of the coefficient estimators are reported for equation (2) only).
- (b) The empirical sizes of the t-ratios for zero coefficient restrictions using a 5% nominal size. Both OLS and heteroscedasticity and autocorrelation consistent (HAC) coefficient standard errors are employed.<sup>11</sup>
- (c) The average coefficients of determination.
- (d) The average regression standard errors.
- (e) The average values of the Durbin-Watson statistic.

We will consider these first for model (8) excluding the  $I(0)$  regressor (with  $\hat{\beta}_1 = 0$ ) and then for model (8).

### 3.1 Simulations for Model (8) excluding the $I(0)$ regressor

Tables 1 and 2 report the averages of  $\tilde{\alpha}$  and  $\tilde{\beta}_2$ , respectively. For both estimators the average values are close to zero, become smaller as the sample size increases (with the possible exception of  $\tilde{\alpha}$  when  $\rho = 1$ ) and are not significantly different from zero in virtually all cases.<sup>12</sup> However, because positive and negative values of these coefficients will likely be cancelling each other out when averaged we consider the average magnitudes of these coefficients in Tables 3 and 4. When  $\rho < 1$  there is a clear tendency for the coefficients to tend to zero asymptotically where this convergence is faster for smaller values of  $\rho$  relative to larger values.<sup>13</sup> In contrast, when  $\rho = 1$  the average magnitude of the intercept increases as the sample size grows

<sup>10</sup> These are the sample sizes after the first 500 observations have been discarded.

<sup>11</sup> For the HAC standard errors we employ the Bartlett window,  $\left[1 - \left(\frac{h}{g+1}\right)\right]$ , where  $h = 1, 2, \dots, g$  and

$g$  is set equal the integer component of  $4\left(\frac{T}{100}\right)^{2/9}$ . This is a widely used option that is available in

many econometric software packages (it is the default option in EViews 5.1).

<sup>12</sup> For  $\tilde{\alpha}$  there are 5 rejections of the null out of 88 (giving an empirical size of 5.9%) using a 5% nominal size while for  $\tilde{\beta}_2$  there are 3 rejections out of 88 (3.4%). With the empirical sizes being close to the 5% nominal size this is consistent with a small percentage of chance rejections. Thus, the rejection rate is consistent with a general acceptance of the null that each coefficient is individually zero.

<sup>13</sup> For small samples some of the average magnitudes are relatively large, especially for greater values of  $\rho$ . We have not calculated tests of whether the absolute values of the coefficients are significantly greater than zero (requiring the standard errors of the coefficients' magnitudes), nevertheless, the tendencies are clear from the results reported in Tables 3 and 4.

while the mean absolute value of  $\tilde{\beta}_2$  remains constant at a non-zero value.<sup>14</sup> Nevertheless, it is clear that both coefficients converge to zero asymptotically when the dependent variable is stationary, which confirms the asymptotic theory cited above.

The empirical sizes (given a 5% nominal size) of  $t_{\tilde{\alpha}}$  and  $t_{\tilde{\beta}_2}$ , using OLS coefficient standard errors, are reported in Table 5 and 7, respectively. For both t-ratios the empirical size is insignificantly different from 5% when  $\rho = 0$  and significantly different from the nominal size for  $\rho > 0$ . Hence, there is evident spurious significance when  $\rho > 0$ . The degree of size distortion appears to converge to a constant (for  $0 \leq \rho < 1$ ) as the sample size rises, which is consistent with both t-ratios converging to a random variable and in line with the asymptotic results cited above.<sup>15</sup> Further, the degree of size distortion rises as the value of  $\rho$  increases. For example, the empirical size of both t-ratios is in the range of 7.18% to 7.82% when  $\rho = 0.1$ , ranges from 19.40% to 26.18% for  $\rho = 0.5$  and with  $\rho = 0.9$  takes on values between 44.89% and 65.84%. Thus, the degree of spurious correlation exhibited by the coefficient t-ratios can become quite severe and does not diminish in large samples.

The empirical sizes for these t-ratios, using HAC coefficient standard errors ( $t_{\tilde{\alpha}}^{hac}$  and  $t_{\tilde{\beta}_2}^{hac}$ ), are given in Table 6 and 8, respectively. As expected the consistency inducing effect lowers the empirical sizes asymptotically, provided  $\rho < 1$ .<sup>16</sup> While the reduction in the empirical size is generally quite substantial there still remains statistically significant size distortion, even in large samples. For example when  $T = 10,000$  the empirical sizes are 6.52% for  $t_{\tilde{\alpha}}^{hac}$  and 7.02% for  $t_{\tilde{\beta}_2}^{hac}$  when  $\rho = 0.5$ , 8.47% and 8.53% when  $\rho = 0.7$  and 19.64% and 19.57% when  $\rho = 0.9$ . Thus, whilst the use of HAC coefficient standard errors reduces spurious correlation it does not eliminate it completely even when the sample size is quite large. Further, the use of HAC standard errors yields less size distortion in both t-ratios compared to using OLS standard errors when  $T \geq 200$  and  $\rho \geq 0.6$  (with the exception of when  $T = 200$  and  $\rho = 0.6$  for  $t_{\tilde{\beta}_2}^{hac}$ ). However, HAC standard errors can increase size distortion in both t-ratios compared to OLS standard errors when both  $T$  and  $\rho$  are relatively small. For example, the empirical sizes of the t-ratios using OLS standard errors are always smaller than those achieved with the HAC adjustment when  $T \leq 100$  and  $\rho \leq 0.1$ . Hence, in certain circumstances the use of HAC coefficient standard errors will not improve inference relative to t-ratios based on standard OLS coefficient standard errors.

<sup>14</sup> The results with  $\rho = 1$  are consistent with Phillips (1986) who shows that, in the regression of an  $I(1)$  variable on a constant and an  $I(1)$  regressor, the intercept will diverge to infinity and the slope coefficient will converge to a random variable asymptotically.

<sup>15</sup> The results with  $\rho = 1$  indicate that the size distortion of both t-statistics increases asymptotically, which is consistent with Phillips (1986) who shows that, in the regression of an  $I(1)$  variable on a constant and an  $I(1)$  regressor, both intercept and the slope coefficient's t-ratios will diverge to infinity asymptotically.

<sup>16</sup> When  $\rho = 1$  the degree of size distortion still rises asymptotically for both t-ratios, despite the HAC correction.

Table 9 reports the average values of the coefficient of determination. Although the average  $\tilde{R}^2$  is significantly different from zero for all  $T$  and  $\rho$  it clearly tends to zero asymptotically when  $\rho < 1$ .<sup>17</sup> Numerically the average coefficient of determination is 0.00 (rounded to two decimal places) for all  $0 \leq \rho \leq 0.9$  when  $T \geq 5,000$ . This is sufficiently small to be consistent with the prediction from asymptotic theory (cited above) that  $\tilde{R}^2$  tends to zero in large samples.<sup>18</sup> Hence, the coefficient of determination does not exhibit spurious regression asymptotically when the dependent variable is  $I(0)$  because it converges to its expected value of zero. There may be some small exaggeration of  $\tilde{R}^2$  in small samples.

Averages of the regression standard error,  $\tilde{s}$ , are reported in Table 10. The far right column of this table (headed  $\sigma_u$ ) gives the true regression standard error, for each value of  $\rho$  (for all  $T$ ) when  $\rho < 1$ , which is calculated as the square root of (3), thus,

$$\sigma_u = +\sqrt{\frac{1}{1-\rho^2}}.^{19}$$

With the exception of a small number of terms (for small  $\rho$  and large  $T$ ) the averages of  $\tilde{s}$  are significantly different from their true values,  $\sigma_u$ . However, for  $\rho < 1$  the averages of  $\tilde{s}$  clearly tend towards  $\sigma_u$  asymptotically. Indeed, for  $T = 10,000$  these numbers are the same (to three decimal places) except when  $\rho = 0.5$  ( $\tilde{s} = 1.154$  and  $\sigma_u = 1.155$  giving an absolute difference of  $|\tilde{s} - \sigma_u| = 0.001$ ),  $\rho = 0.7$  ( $|\tilde{s} - \sigma_u| = 0.001$ ),  $\rho = 0.8$  ( $|\tilde{s} - \sigma_u| = 0.002$ ) and  $\rho = 0.9$  ( $|\tilde{s} - \sigma_u| = 0.005$ ). This is consistent with the asymptotic theory cited above in the respect that the average regression standard error (variance) is converging to a random variable [provided that the dependent variable is  $I(0)$ ].<sup>20</sup> It also suggests that this statistic does not exhibit spurious regression when  $\rho < 1$  because it converges to its true value asymptotically.<sup>21 22</sup>

<sup>17</sup> When  $\rho = 1$  the coefficient of determination does not converge to zero asymptotically, indeed,  $\tilde{R}^2 = 0.24$  for all sample sizes. This is consistent with Phillips (1986) who shows that the coefficient of determination in the regression of an  $I(1)$  variable on a constant and an  $I(1)$  regressor will converge to a random variable asymptotically.

<sup>18</sup>  $\tilde{R}^2$  can have moderate values in small samples. For example, when  $T = 25$  and  $\rho = 0.9$ ,  $\tilde{R}^2 = 0.19$ .

<sup>19</sup> When  $\rho = 1$ , the true values of  $\tilde{s}$  for each  $T$  are given in italics in the bottom row of Table 10. They are calculated with  $\sigma_u = +\sqrt{T}$ .

<sup>20</sup> In contrast, when  $\rho = 1$  the average  $\tilde{s}$  diverges to infinity asymptotically. This is consistent with results reported (although not commented upon) in Phillips (1986, p. 331) which show that the error variance (standard error of the regression) diverges at rate  $T$  (diverges at rate  $\sqrt{T}$ ) in the regression of an  $I(1)$  variable on a constant and an  $I(1)$  regressor.

<sup>21</sup> Although the average  $\tilde{s}$  diverges to infinity asymptotically when  $\rho = 1$ , and does so at the same rate as the true standard error ( $\sigma_u$ ), it systematically underpredicts  $\sigma_u$ , therefore, indicating exaggerated fit (spurious regression). This is because the random variable that  $T^{-1}\tilde{s}^2$  converges to (as reported in Phillips, 1986, p. 331), must have a below unit mean (given  $\sigma_{e_0}^2 = 1$ ), since  $T^{-1}\sigma_u^2$  is unity – see the appendix.

Table 11 reports the average values of the Durbin-Watson statistic. The asymptotic values predicted by the theory cited above are given in the far right column for each value of  $\rho$ . Although virtually all of the averages are significantly different from their predicted asymptotic values there is a clear tendency towards these values as the sample size increases. Indeed, the numerical differences between the simulated values of  $\tilde{d}$  when  $T = 10,000$  and the true asymptotic values do not exceed 0.002 for any  $\rho$ . Hence, the numerical difference is extremely small. Thus, these results are interpreted as being consistent with the theoretical results cited above.

The empirical sizes of  $\tilde{d}$  are given in Table 12. All but one are significantly different from the 5% nominal size. A small number of the empirical sizes are below 5% (when  $\rho = 0$  and when  $\rho = 0.1$  and  $T = 10,000$ ). These undersized values are likely due to the use of the lower bound of the DW critical values to calculate the rejection rates, where the *true* critical values would be between the upper and lower bound. However, the vast majority of results indicate that  $\tilde{d}$  is excessively oversized (the rejection rate is often 100%). Further, the empirical size rises both with the sample size and the autocorrelation coefficient. Thus, except for when  $\rho = 0$  the model is virtually always subject to residual autocorrelation.

### 3.2 Simulations for Model (8) including the $I(0)$ regressor

For the three-variable model, equation (8), simulations there are four dimensions to the results. That is, the simulated values are calculated for various values of  $T$ ,  $\rho$  and  $\phi$ . As for the two-variable model simulations, the tables are presented in the following three dimensions: simulated values of a statistic,  $T$  and  $\rho$ . Thus, for each statistic (coefficient, t-ratio etc.) there are 11 different tables with each corresponding to the different 11 values that  $\phi$  can take on ( $\phi = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$ ). To reduce the number of tables to be presented we consider whether the results are similar across the different values of  $\phi$ . Specifically, we consider whether the simulated values (which are averages) for each particular combination of  $T$  and  $\rho$  are significantly different for every pair of  $\phi$  values.<sup>23</sup> This employs the standard test for the difference in two random variables'

<sup>22</sup> This statistic may exhibit some spurious regression in small samples because the OLS estimator of the regression standard error is known to be downward biased if both the residuals and the explanatory variables are positively autocorrelated (as they are in this example). This is reflected in the convergence towards the true value from below in Table 10.

<sup>23</sup> For example, the test is conducted for the pairings:  $\phi = 0.0$  and  $\phi = 0.1$ ,  $\phi = 0.0$  and  $\phi = 0.2, \dots, \phi = 0.9$  and  $\phi = 1.0$ . There are a maximum of 55 pairings (which could be viewed as samples). Within these pairings/samples there are up to 88 tests corresponding to the different combinations of  $T$  and  $\rho$ . If a nominal size of the difference in means test is set at 5% one would expect  $(0.05 \times 88 =)$  4.4 rejections of the null per pairing/sample if the null is true. This expectation is used in a chi-square test for whether the observed number of rejections in sample  $j$ , denoted  $o_j$ , is different

from the expected number of rejections,  $e_j$ . The test statistic and distribution is  $\sum_{j=1}^J \frac{(o_j - e_j)^2}{e_j} \sim \chi_{J-1}^2$ ,

where  $J$  is the number of samples/pairings used in the calculation.

means being significantly different from zero.<sup>24</sup> Because there are a large number of these tests we further test whether the percentage of rejections (empirical size) is equal to the nominal size (set at 5%) using the standard chi-square test for equality of proportions (percentages) across samples (where each sample corresponds to each pair of  $\phi$  values).

Table 13 reports the percentage of rejections of the difference in means test (in the column headed “Empirical size”), the chi-square test statistic (in the column headed “ $\chi^2(v)$ ”, where  $v$  denotes degrees of freedom) and its associated probability value (in the column headed “p-value”). Three sets of results are reported. The first includes all tests (entitled “Including  $\rho = 1$  and  $\phi = 1$ ”) while the second includes all tests except those where either  $\rho = 1$  or  $\phi = 1$  (headed “Excluding  $\rho = 1$  and  $\phi = 1$ ”). The second test allows only results where  $X_{1t}$  and  $Y_{0t}$  are  $I(0)$ , which is the case that is the focus of this paper. The third set of results includes all tests except those where  $\rho = 1$ ,  $\rho = 0.9$ ,  $\phi = 1$  or  $\phi = 0.9$  (entitled “Excluding  $\rho = 1$ ,  $\rho = 0.9$ ,  $\phi = 1$  and  $\phi = 0.9$ ”) to exclude results where  $X_{1t}$  and  $Y_{0t}$  contain unit roots or near unit roots. For all statistics (magnitudes of coefficients, empirical sizes of t-ratios, averages of fit measures and Durbin-Watson statistics) the chi-square test is rejected, indicating that the results are significantly different for different values of  $\phi$ , with two exceptions. The two exceptions are for the empirical sizes of t-ratios for the intercept and  $I(1)$  regressor (the sizes are 4.82% and 4.63%, respectively) when the tests exclude results where  $\rho = 1$ ,  $\rho = 0.9$ ,  $\phi = 1$  or  $\phi = 0.9$ . In these two cases the results are not significantly different for all  $\phi$ , excepting  $\phi = 0.9$  and  $\phi = 1$ . This finding will be used to simplify the summary of results for the three-variable LRM.

The average magnitudes of  $\hat{\alpha}$ ,  $\hat{\beta}_1$  and  $\hat{\beta}_2$  are reported in Tables 14a to 14d, 15a to 15d and 16a to 16d, respectively. The four tables for each coefficient correspond to each of the following four values of  $\phi$ :  $\phi = 0.0$ ,  $\phi = 0.5$ ,  $\phi = 0.9$  and  $\phi = 1$ . For each coefficient estimator there is a clear tendency for the coefficients to move towards zero asymptotically when  $\rho < 1$ . This convergence is faster for smaller values of  $\rho$  relative to larger values.<sup>25</sup> In contrast, when  $\rho = 1$  the average magnitude of the intercept increases as the sample size rises while the mean absolute value of the slope coefficients remain constant at non-zero values.<sup>26</sup> There are some minor variations in the rates of convergence and divergence for different values of  $\phi$ , however, the

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<sup>24</sup> The standard test statistic is  $Z = \frac{\bar{W}_1 - \bar{W}_2}{\sqrt{\frac{s_{W1}^2}{n_{W1}} + \frac{s_{W2}^2}{n_{W2}}}}$ , with,  $s_{W_i}^2 = \frac{W_i - \bar{W}_i}{n_{W_i} - 1}$ , where  $W_i$  is the value of the

statistic calculated for each replication in the  $i^{\text{th}}$  sample/pairing and  $\bar{W}_i$  is the mean value of  $W_i$  calculated over the ( $n_{W1} = n_{W2} =$ ) 10,000 replications. According to the Central Limit Theorem this test statistic is normally distributed for a large number of replications (as in our example).

<sup>25</sup> These results are similar to those reported for the magnitudes of coefficients in the two-variable LRM, reported in Tables 3 and 4.

<sup>26</sup> The results with  $\rho = 1$  are consistent with Phillips (1986) who shows that, in the regression of an  $I(1)$  variable on a constant and a set of  $I(1)$  regressors, the intercept will diverge to infinity and the slope coefficients will converge to random variables asymptotically.

qualitative results remain. Overall it is clear that all three coefficients converge to zero asymptotically when the dependent variable is stationary, which confirms the asymptotic theory cited above.

The empirical sizes (given a 5% nominal size) of t-ratios, using OLS coefficient standard errors, are reported for up to the following four values of  $\phi$ :  $\phi = 0.0$ ,  $\phi = 0.5$ ,  $\phi = 0.9$  and  $\phi = 1$ . Tables 17a to 17c report empirical sizes of  $t_{\hat{\alpha}}$  for three values of  $\phi$  - no table is provided for  $\phi = 0.0$ .<sup>27</sup> The rejection rates of the  $I(0)$  regressor's t-ratio ( $t_{\hat{\beta}_1}$ ) are given for all four values of  $\phi$  in Tables 19a to 19d, while Table 21a to 21c report the empirical sizes for the  $I(1)$  variable's t-statistic ( $t_{\hat{\beta}_2}$ ) for three values of  $\phi$  - no results are provided for  $\phi = 0.0$ . For all three coefficient t-ratios and all values of  $\phi$  the empirical sizes are insignificantly different from 5% when  $\rho = 0$  and significantly different from the nominal size when  $\rho > 0$  [with the exception of the  $I(0)$  regressor's t-statistic for  $\phi = 0.0$ , where all rejection rates are insignificantly different from the 5% nominal size]. Hence, there is evident spurious significance when  $\rho > 0$  in virtually all cases. The rejection rates appear to converge to constants (for  $0 \leq \rho < 1$ ) as the sample size increases, which is consistent with all three t-ratios converging to a random variable and in line with the asymptotic results cited above.<sup>28</sup> Further, the degree of size distortion rises as the value of  $\rho$  increases and can be large. For the intercept and the  $I(1)$  regressor's t-statistics the empirical sizes remain relatively constant for different values of  $\phi$ , however, the rejection rates rise as  $\phi$  increases for the  $I(0)$  variable's t-ratio. In general, the degree of spurious significance exhibited by the coefficient t-ratios of all the explanatory variables can become quite severe and does not diminish in large samples.

The empirical sizes (with a 5% nominal size) of t-ratios, using HAC coefficient standard errors, are also reported for the same values of  $\phi$  as before. The values for  $t_{\hat{\alpha}}$  are reported in Table 18a to 18c,  $t_{\hat{\beta}_1}$  sizes are reported in Table 20a to 20d and  $t_{\hat{\beta}_2}$  are given in Table 22a to 22c. For all three t-statistics the rejection rates decline asymptotically when  $\rho < 1$ . This is consonant with the consistency property embodied in the HAC standard errors – although not many empirical sizes are significantly different from 5% when the sample size is 10,000.<sup>29</sup> In small samples and with small

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<sup>27</sup> From Table 13 it was discovered that the results for all values of  $\phi \leq 0.8$  were not significantly different for the t-ratios' empirical sizes of the intercept and  $I(1)$  regressor. Hence, the results are only reported for one representative value,  $\phi = 0.5$  when  $\phi \leq 0.8$ .

<sup>28</sup> The results with  $\rho = 1$  indicate that the size distortion of the t-statistics for the intercept and  $I(1)$  slope coefficient increase asymptotically, which is consistent with Phillips (1986). However, for the  $I(0)$  regressor's t-ratio when  $\rho = 1$  the asymptotic behaviour depends on the value of  $\phi$ . The  $I(0)$  regressor's t-ratio converges to a constant of approximately 5% when  $\phi = 0.0$ , to a constant of about 26% when  $\phi = 0.5$ , to a constant of about 65% when  $\phi = 0.9$  and diverges to infinity when  $\phi = 1$ . The result for  $\phi = 1$  is consistent with Phillips (1986) and the results for  $\phi = 0.0$ ,  $\phi = 0.5$  and  $\phi = 0.9$  are consistent with Stewart (2006).

<sup>29</sup> The results with  $\rho = 1$  indicate that the size distortion of the HAC adjusted t-statistics for the intercept and  $I(1)$  slope coefficient increase asymptotically. In contrast, the rejection rates of the  $I(0)$

values of  $\rho$  the empirical sizes of the t-statistics using HAC standard errors are often larger than t-ratios calculated with OLS standard errors. However, rejection rates of t-ratios based on HAC standard errors are generally smaller than those obtained with OLS standard errors.

The average values of the coefficient of determination are reported in Tables 23a to 23d for  $\phi = 0.0$ ,  $\phi = 0.5$ ,  $\phi = 0.9$  and  $\phi = 1$ . The average  $R^2$  is significantly different from zero for all values of  $T$  and  $\rho$ , however, when  $\rho < 1$  it clearly tends to zero asymptotically.<sup>30</sup> Indeed, the average coefficient of determination is 0.00 (rounded to two decimal places) for all  $0 \leq \rho \leq 0.9$  when  $T = 10,000$ . This is sufficiently small to be consistent with the prediction from asymptotic theory (cited above) that  $R^2$  tends to zero in large samples. Hence, the coefficient of determination does not exhibit spurious regression asymptotically when the dependent variable is  $I(0)$  because it converges to its expected value of zero. There may be some exaggeration of  $R^2$  in small samples.<sup>31</sup>

Tables 24a to 24d report the averages of the regression standard error,  $s$ , for  $\phi = 0.0$ ,  $\phi = 0.5$ ,  $\phi = 0.9$  and  $\phi = 1$ . The far right column of these tables (headed  $\sigma_u$ ) give the true regression standard error, for each value of  $\rho$  (for all  $T$ ) when  $\rho < 1$ , which is calculated as the square root of (3), thus,  $\sigma_u = +\sqrt{\frac{1}{1-\rho^2}}$ .<sup>32</sup> With the exception of a small number of terms (for small  $\rho$  and large  $T$ ) the averages of  $s$  are significantly different from their true values,  $\sigma_u$ . However, the averages of  $s$  clearly tend towards  $\sigma_u$  asymptotically when  $\rho < 1$ . Indeed,  $s$  and  $\sigma_u$  are generally the same (to three decimal places) when  $T = 10,000$ . The *largest* absolute difference between these two values is  $|s - \sigma_u| = 0.007$ , which occurs when  $\rho = 0.9$ ,  $\phi = 1$  and  $T = 10,000$ . This is consistent with the asymptotic theory cited above in the respect that the average regression standard error (variance) is converging to a random variable [provided that the dependent variable is  $I(0)$ ].<sup>33</sup> The results also suggest that  $s$  does not exhibit

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regressor's HAC adjusted t-ratio decline as the sample size increases when  $\rho = 1$ , except for  $\phi = 1$ , when the empirical size diverges to infinity.

<sup>30</sup> When  $\rho = 1$  the coefficient of determination does not converge to zero asymptotically. For  $\phi < 1.0$   $R^2$  tends to 0.24 and when  $\phi = 1.0$  and  $R^2$  converges to 0.40. This is consistent with Phillips (1986) who shows that in the regression of an  $I(1)$  variable on a constant and a set of  $I(1)$  regressors the  $R^2$  will converge to a random variable asymptotically.

<sup>31</sup>  $R^2$  can have moderate values in small samples. For example, when  $T = 25$ ,  $\rho = 0.9$  and  $\phi = 0.9$ ,  $R^2 = 0.32$ .

<sup>32</sup> When  $\rho = 1$ , the true values of  $s$  for each  $T$  are given in italics in the bottom row of the tables. They are calculated with  $\sigma_u = +\sqrt{T}$ .

<sup>33</sup> In contrast, when  $\rho = 1$  the average  $s$  diverges to infinity asymptotically. This is consistent with results reported (although not commented upon) in Phillips (1986).

spurious regression when  $\rho < 1$  because  $s$  converges to its true value asymptotically.<sup>34 35</sup>

The average values of the Durbin-Watson statistic,  $d$ , are reported in Table 25a to 25d for  $\phi = 0.0$ ,  $\phi = 0.5$ ,  $\phi = 0.9$  and  $\phi = 1$ . The asymptotic values predicted by the theory cited above are given in the far right columns of the tables for each value of  $\rho$ . Although virtually all of the averages are significantly different from their predicted asymptotic values there is a clear tendency towards these large sample values as the sample size increases. Indeed, the numerical *differences* between the simulated values of  $d$  when  $T = 10,000$  and the true asymptotic values do not exceed 0.003 for any  $\rho$ . Hence, the numerical difference is extremely small. Thus, these results are interpreted as being consistent with the theoretical asymptotic results cited above.

Table 26a to 26d give the empirical sizes of  $d$  for  $\phi = 0.0$ ,  $\phi = 0.5$ ,  $\phi = 0.9$  and  $\phi = 1$ . All but two values of  $d$  are significantly different from the 5% nominal size. A small number of the empirical sizes are below 5% (this is always true when  $\rho = 0$  and is occasionally so when  $\rho = 0.1$ ). These undersized values are likely due to the use of the lower bound of the Durbin-Watson critical values to calculate the rejection rates – the *true* critical values would be between the upper and lower bound. However, the vast majority of results indicate that the empirical size of  $d$  is far greater than the 5% nominal size (the rejection rate is often 100%). Further, the empirical size rises both with the sample size and  $\rho$ . Thus, except for when  $\rho = 0$  the model is virtually always subject to residual autocorrelation.

#### 4. Conclusion

This paper has produced simulation evidence to clarify and corroborate asymptotic results regarding various statistics in the OLS regression of an  $I(0)$  dependent variable on an  $I(1)$  regressor, with or without another  $I(0)$  explanatory factor. In both cases, when an  $I(0)$  regressor is included or excluded from the model, the simulation results confirm the asymptotic theory that all coefficients and the coefficient of determination tend to zero: these statistics do not exhibit spurious correlation.<sup>36</sup> The simulation results also clarify that the regression standard error converges to its expected value and, therefore, shows no exaggerated fit (in both cases).

In contrast, the t-ratios of all coefficients generally exhibit significant spurious significance (spurious correlation is not evident only when  $\rho = 0$  in the two-variable case and when  $\rho = 0$  and/or  $\phi = 0.0$  for the three-variable model). The degree of spurious significance rises as  $\rho$  increases for all coefficient t-statistics, however, only becomes more severe when  $\phi$  grows for the  $I(0)$  regressor in the three-variable

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<sup>34</sup> Although the average  $s$  diverges to infinity asymptotically when  $\rho = 1$ , and does so at the same rate as the true standard error ( $\sigma_u$ ), it systematically underpredicts  $\sigma_u$ , therefore, indicating exaggerated fit (spurious regression).

<sup>35</sup> This statistic may exhibit some spurious correlation in small samples because the OLS estimator of the regression standard error is known to be downward biased if both the residuals and the explanatory variables are positively autocorrelated (as they are in this example). This is reflected in the convergence towards the true value from below in the tables.

<sup>36</sup> This summary refers to the findings when  $\rho < 1$  only.

LRM. The degree of spurious correlation can become quite severe for all of the explanatory variables t-ratios. Using the HAC adjustment generally reduces spurious significance in all of the coefficients t-ratios, although does not always eliminate it. That is, spurious significance remains in small samples, however, it tends to be eliminated asymptotically, if very large sample sizes sometimes appear to be required to remove this size distortion.

Finally, the Durbin-Watson statistic generally suggests residual autocorrelation (the exception is when  $\rho = 0$ ). The coincidence of residual autocorrelation and oversized t-ratios is consistent with econometric theory. For example, in the two-variable LRM theory predicts that when, as in our model, both  $X_{2t}$  and the error term are *positively* autocorrelated, the OLS estimator of the variance of  $\beta_2$  will be downward biased.<sup>37</sup> This implies that the t-ratio will be upward biased which, in turn, explains why its empirical size would exceed its nominal size.<sup>38</sup> Because, for this model, the coefficients tend to zero asymptotically, the sole cause of spurious significance is from the denominator of the t-ratio.<sup>39</sup> That is, spurious correlation manifested in coefficient t-ratios is solely caused by residual autocorrelation.

Thus, the results of this paper demonstrate that spurious regression will generally occur in a regression of an  $I(0)$  dependent variable on an  $I(1)$  regressor, with or without another  $I(0)$  regressor. This implies that spurious correlation is more widespread than previously thought. However, spurious correlation is only exhibited by t-ratios and F-statistics for coefficient restrictions and not by any other statistic in this specification.<sup>40</sup> The t- and F-statistics from the model specification considered here (which is of the widely employed error-correction form) can be used to test for cointegration (see, for example, Kanioura and Turner, 2005). This suggests that using currently available critical values may lead to inference of spurious correlation and, therefore, spurious cointegration.

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<sup>37</sup> This downward bias occurs even if there is no bias in the OLS estimation of the residual variance. In our case there is no bias in the regression standard error asymptotically, although the simulation results reported in Table 10 suggest that there is some bias in small samples.

<sup>38</sup> Notice that when  $\rho = 0$  there is no evident autocorrelation (the Durbin-Watson statistic is very close to 2 and its empirical size is near to its nominal size) and no spurious significance exhibited by coefficient t-ratios.

<sup>39</sup> Although the coefficient is zero on average it is not generally zero. That is, the coefficient is non-zero in most samples, which allows the downward bias in the t-ratio's denominator to raise the t-statistic above its critical value in a greater proportion of samples than suggested by the nominal size of the test.

<sup>40</sup> The results for zero coefficient restrictions based on the t-ratios directly generalise to F-statistics because the F-statistic will give exactly the same inference as provided by the t-ratio for single coefficient restrictions.

### Appendix: the expected population variance

Given the following population model corresponding to the sample regression function, model (1), and encompassing model (2):

$$Y_{0t} = \alpha + \underline{\beta}_1' \underline{X}_{1t} + \underline{\beta}_2' \underline{X}_{2t} + u_t \quad (\text{A.1})$$

The error term is:

$$u_t = Y_{0t} - \alpha - \underline{\beta}_1' \underline{X}_{1t} - \underline{\beta}_2' \underline{X}_{2t} \quad (\text{A.2})$$

If all the variables in the model are independently generated, the slope coefficients will equal zero. In this case, the error term becomes:

$$u_t = Y_{0t} - \alpha \quad (\text{A.3})$$

The variance of the error term is:

$$\text{Var}(u_t) = \text{Var}(Y_{0t} - \alpha) = E[Y_{0t} - \alpha - E(Y_{0t} - \alpha)]^2 = E[Y_{0t} - E(Y_{0t})]^2 \quad (\text{A.4})$$

If the true DGP of the dependent variable is:

$$Y_{0t} = v_{0t}, \quad \text{where,} \quad v_{0t} = \rho v_{0t-1} + e_{0t}, \quad -1 < \rho < 1 \quad (\text{A.5})$$

Then, given stationarity, the mean and variance of this process are:

$$E(Y_{0t}) = 0 \quad (\text{A.6})$$

$$\text{Var}(Y_{0t}) = \left( \frac{1}{1 - \rho^2} \right) \sigma_{e_0}^2 \quad (\text{A.7})$$

Substitution of (A.6) and (A.7) into (A.4) gives:

$$\text{Var}(u_t) = E[Y_{0t} - E(Y_{0t})]^2 = E[Y_{0t}]^2 = \text{Var}(Y_{0t}) = \left( \frac{1}{1 - \rho^2} \right) \sigma_{e_0}^2 \quad (\text{A.8})$$

If, as in our Monte Carlo simulation experiment,  $\sigma_{e_0}^2 = 1$  then:

$$\sqrt{\text{Var}(u_t)} = \sqrt{\left( \frac{1}{1 - \rho^2} \right)} \quad (\text{A.9})$$

Thus, in our Monte Carlo simulation experiment, (A.9) is the population value that the unbiased estimator of the regression standard error should converge to (when  $Y_{0t}$  is

stationary). Similarly, (A.8) is the population variance that the unbiased estimator of the regression variance should converge to.

When  $Y_{0t} \sim I(1)$  the *true* variance should be (see, for example, Gujarati, 2003, p. 799):

$$\text{Var}(u_t) = \text{Var}(Y_{0t}) = T\sigma_{e_0}^2 \quad (\text{A.10})$$

If, as in our Monte Carlo simulation experiment,  $\sigma_{e_0}^2 = 1$  then:

$$\text{Var}(u_t) = \text{Var}(Y_{0t}) = T \quad (\text{A.11})$$

This implies that:

$$\sqrt{\text{Var}(u_t)} = \sqrt{T} \quad (\text{A.12})$$

## References

- Durlauf, S. N. and Phillips, P. C. B. (1988), "Trends versus Random Walks in Time Series Analysis", *Econometrica*, 56, 6, 1333 – 1354.
- Entorf, H. (1997), "Random Walks with Drifts: Nonsense Regression and Spurious Fixed-Effect Estimation", *Journal of Econometrics*, 80, 287 – 296.
- Granger, C. W J, Namwon Hyung and Jeon Yongil (2001) "Spurious Regressions with Stationary Series", *Applied Economics*, 33, 899 – 904.
- Granger, C. W. J. and Newbold, P. (1974), "Spurious Regressions in Econometrics", *Journal of Econometrics*, 2, 111 – 120.
- Gujarati, D N. (2003), *Basic Econometrics*, McGraw Hill, (4<sup>th</sup> edition).
- Haldrup, N. (1994), "The Asymptotics of Single-Equation Cointegration Regressions with  $I(1)$  and  $I(2)$  Variables", *Journal of Econometrics*, 63, 153 – 181.
- Hassler, U. (1996), "Spurious Regressions when Stationary Regressors are Included", *Economics Letters*, 50, 25 – 31.
- Hassler U. (2003), "Nonsense Regressions due to Neglected Time-Varying Means", *Statistical Papers*, 44, 169 – 182.
- Kanioura A. and Turner P. (2005), "Critical Values for an F-test for Cointegration in a Multivariate Model", *Applied Economics*, 37, 265 – 270.
- Kim T. Lee Y. and Newbold P. (2004), "Spurious Regressions with Stationary Processes around Linear Trends", *Economics Letters*, 83, 257 – 262.
- Marmol F. (1995), "Spurious Regressions Between  $I(d)$  Processes", *Journal of Time Series Analysis*, 16, 3, 313 – 321.
- Marmol F. (1996), "Nonsense Regressions between Integrated Processes of Different Orders", *Oxford Bulletin of Economics and Statistics*, 58, 3, 525 – 536.
- Marmol F. (1998), "Spurious Regression Theory with Nonstationary Fractionally Integrated Processes", *Journal of Econometrics*, 84, 233 – 250.
- Nelson C. R. and Kang H. (1984), "Pitfalls in the use of Time as an Explanatory Variable in Regression", *Journal of Business and Economic Statistics*, 2, 73 – 82.
- Phillips P. C. B. (1986), "Understanding Spurious Regressions in Econometrics", *Journal of Econometrics*, 33, 311 – 340.
- Stewart C. (2006), "Spurious Correlation of  $I(0)$  Regressors in Models with an  $I(1)$  Dependent Variable", *Economics Letters*, 91, 184 – 189.

Stewart C. (2007), “Spurious correlation of  $I(1)$  regressors in models with an  $I(0)$  dependent variable: asymptotic results”, Department of Economics Discussion Paper, London Metropolitan University. Downloadable at: [http://www.londonmet.ac.uk/londonmet/library/g21746\\_3.pdf](http://www.londonmet.ac.uk/londonmet/library/g21746_3.pdf)

Sun Y. (2006), “Spurious Regressions between Stationary Generalized Long Memory Processes”, *Economics Letters*, 90, 446 – 454.

Tsay W. and Chung C. (2000), “The Spurious Regression of Fractionally Integrated Processes”, *Journal of Econometrics*, 96, 155 – 182.

**Table 1: Averages of  $\tilde{\alpha}$  in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.01 (0.676)	0.01 (0.613)	0.00 (0.908)	0.00 (0.859)	0.00 (0.451)	0.00 (0.116)	0.00 (0.148)	0.00 (0.694)
<b>0.10</b>	-0.00 (0.894)	0.03 (0.059)*	-0.01 (0.108)	-0.00 (0.536)	-0.00 (0.756)	0.00 (0.418)	0.00 (0.337)	-0.00 (0.945)
<b>0.20</b>	-0.06 (0.078)*	0.03 (0.076)*	-0.01 (0.142)	-0.01 (0.175)	-0.00 (0.564)	-0.00 (0.625)	-0.00 (0.272)	0.00 (0.291)
<b>0.30</b>	-0.07 (0.052)*	0.01 (0.726)	-0.01 (0.588)	0.00 (0.937)	-0.00 (0.369)	0.00 (0.460)	0.00 (0.367)	-0.00 (0.045)**
<b>0.40</b>	0.01 (0.867)	0.01 (0.724)	0.01 (0.384)	0.00 (0.516)	-0.00 (0.944)	0.00 (0.906)	0.00 (0.364)	0.00 (0.931)
<b>0.50</b>	0.02 (0.689)	0.03 (0.297)	-0.00 (0.967)	0.00 (0.520)	0.00 (0.970)	-0.00 (0.655)	0.00 (0.760)	0.00 (0.942)
<b>0.60</b>	0.04 (0.459)	-0.01 (0.851)	0.01 (0.422)	-0.00 (0.675)	-0.01 (0.004)***	0.00 (0.239)	-0.00 (0.021)**	-0.00 (0.590)
<b>0.70</b>	0.02 (0.810)	0.00 (0.966)	-0.01 (0.543)	-0.01 (0.657)	0.00 (0.589)	-0.00 (0.188)	-0.00 (0.293)	0.00 (0.231)
<b>0.80</b>	-0.02 (0.805)	0.02 (0.707)	-0.02 (0.436)	-0.03 (0.139)	-0.01 (0.062)*	-0.00 (0.438)	-0.00 (0.318)	-0.00 (0.345)
<b>0.90</b>	-0.12 (0.257)	0.08 (0.290)	0.08 (0.125)	0.02 (0.456)	-0.00 (0.893)	0.01 (0.467)	0.01 (0.025)**	0.00 (0.095)*
<b>1.00</b>	-0.10 (0.706)	-0.35 (0.191)	-0.21 (0.441)	-0.14 (0.620)	-0.17 (0.554)	0.65 (0.033)**	-0.06 (0.896)	0.28 (0.609)

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The averages of  $\tilde{\alpha}$  are reported

P-values of t-tests for the averages of  $\tilde{\alpha}$  equalling zero are given in parentheses

\*, \*\* and \*\*\* denote the averages of  $\tilde{\alpha}$  are significantly different from zero at the 10%, 5% and 1% levels, respectively

**Table 2: Averages of  $\tilde{\beta}_2$  in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	-0.00 (0.418)	-0.00 (0.071)*	0.00 (0.628)	0.00 (0.561)	0.00 (0.945)	0.00 (0.941)	0.00 (0.463)	0.00 (0.210)
<b>0.10</b>	-0.00 (0.176)	-0.00 (0.214)	0.00 (0.899)	0.00 (0.816)	0.00 (0.387)	0.00 (0.617)	0.00 (0.255)	0.00 (0.775)
<b>0.20</b>	0.00 (0.963)	-0.00 (0.100)*	0.00 (0.837)	0.00 (0.599)	-0.00 (0.511)	-0.00 (0.628)	0.00 (0.135)	-0.00 (0.353)
<b>0.30</b>	0.00 (0.143)	0.00 (0.653)	-0.00 (0.449)	-0.00 (0.018)**	0.00 (0.573)	-0.00 (0.278)	0.00 (0.293)	0.00 (0.613)
<b>0.40</b>	-0.00 (0.697)	0.00 (0.938)	0.00 (0.464)	-0.00 (0.073)*	-0.00 (0.055)*	-0.00 (0.809)	-0.00 (0.586)	-0.00 (0.561)
<b>0.50</b>	-0.00 (0.609)	0.00 (0.832)	-0.00 (0.891)	-0.00 (0.798)	0.00 (0.924)	-0.00 (0.750)	-0.00 (0.548)	-0.00 (0.964)
<b>0.60</b>	0.00 (0.112)	0.00 (0.846)	0.00 (0.729)	-0.00 (0.572)	0.00 (0.537)	-0.00 (0.990)	-0.00 (0.399)	0.00 (0.614)
<b>0.70</b>	-0.00 (0.442)	0.00 (0.581)	-0.00 (0.575)	0.00 (0.258)	-0.00 (0.123)	-0.00 (0.287)	0.00 (0.794)	0.00 (0.877)
<b>0.80</b>	0.00 (0.916)	0.00 (0.497)	-0.00 (0.246)	-0.00 (0.285)	-0.00 (0.267)	-0.00 (0.757)	-0.00 (0.157)	-0.00 (0.128)
<b>0.90</b>	-0.00 (0.807)	-0.01 (0.108)	0.00 (0.406)	-0.00 (0.645)	0.00 (0.639)	0.00 (0.705)	-0.00 (0.827)	0.00 (0.390)
<b>1.00</b>	0.01 (0.432)	0.01 (0.029)**	-0.00 (0.839)	-0.01 (0.027)**	-0.00 (0.463)	0.01 (0.204)	-0.01 (0.107)	-0.01 (0.403)

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The averages of  $\tilde{\beta}_2$  are reported

P-values of t-tests for the averages of  $\tilde{\beta}_2$  equalling zero are given in parentheses

\*, \*\* and \*\*\* denote the averages of  $\tilde{\beta}_2$  are significantly different from zero at the 10%, 5% and 1% levels, respectively

**Table 3: Magnitudes of  $\tilde{\alpha}$  in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	1.82	0.89	0.45	0.24	0.11	0.06	0.02	0.01
<b>0.10</b>	2.00	0.99	0.51	0.27	0.12	0.07	0.02	0.02
<b>0.20</b>	2.11	1.07	0.57	0.30	0.13	0.08	0.03	0.02
<b>0.30</b>	2.32	1.23	0.64	0.34	0.15	0.09	0.03	0.02
<b>0.40</b>	2.57	1.38	0.74	0.39	0.18	0.10	0.03	0.02
<b>0.50</b>	2.88	1.62	0.88	0.48	0.21	0.12	0.04	0.03
<b>0.60</b>	3.41	1.94	1.09	0.58	0.27	0.15	0.05	0.03
<b>0.70</b>	4.06	2.46	1.35	0.78	0.36	0.21	0.07	0.05
<b>0.80</b>	5.06	3.28	1.99	1.12	0.52	0.31	0.10	0.07
<b>0.90</b>	6.61	5.05	3.28	2.03	1.00	0.61	0.21	0.14
<b>1.00</b>	20.91	20.89	20.98	21.49	22.11	23.84	33.55	42.89

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\tilde{\alpha}$  are reported

**Table 4: Magnitudes of  $\tilde{\beta}_2$  in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.10	0.05	0.02	0.01	0.00	0.00	0.00	0.00
<b>0.10</b>	0.11	0.05	0.03	0.01	0.01	0.00	0.00	0.00
<b>0.20</b>	0.12	0.06	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.30</b>	0.13	0.07	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.40</b>	0.14	0.08	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.50</b>	0.16	0.09	0.05	0.02	0.01	0.00	0.00	0.00
<b>0.60</b>	0.19	0.10	0.06	0.03	0.01	0.01	0.00	0.00
<b>0.70</b>	0.22	0.13	0.07	0.04	0.02	0.01	0.00	0.00
<b>0.80</b>	0.28	0.18	0.10	0.06	0.02	0.01	0.00	0.00
<b>0.90</b>	0.36	0.27	0.17	0.10	0.04	0.02	0.00	0.00
<b>1.00</b>	0.49	0.48	0.49	0.49	0.49	0.49	0.49	0.49

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The magnitudes of  $\tilde{\beta}_2$  are reported

**Table 5: Empirical sizes of  $t_{\tilde{\alpha}}$  (for  $H_0 : \alpha = 0$ ) in the two-variable LRM:  
OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.25%	4.79%	4.74%	5.41%	4.97%	4.67%	4.84%	4.84%
	(0.262)	(0.326)	(0.221)	(0.070)*	(0.890)	(0.118)	(0.456)	(0.456)
<b>0.10</b>	7.18%	7.31%	7.10%	7.67%	7.82%	7.56%	7.61%	7.66%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	9.38%	10.23%	11.20%	10.46%	10.69%	11.03%	11.41%	11.53%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	12.86%	13.98%	14.49%	14.72%	15.14%	15.13%	14.30%	14.97%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	16.48%	18.02%	18.48%	20.10%	20.45%	19.41%	19.98%	19.59%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	20.03%	23.01%	25.04%	25.58%	25.23%	25.85%	26.01%	26.18%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	26.02%	28.88%	31.18%	32.31%	32.53%	32.99%	33.28%	32.33%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	32.93%	37.06%	38.76%	40.10%	41.37%	40.74%	41.20%	39.95%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	40.35%	45.92%	49.43%	50.24%	50.87%	51.36%	49.96%	50.90%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	48.74%	57.84%	61.18%	63.61%	64.78%	65.45%	65.50%	65.55%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	79.11%	85.85%	90.01%	92.51%	94.80%	96.31%	98.07%	98.26%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The empirical sizes (in percentages) of  $t_{\tilde{\alpha}}$  (using a 5% nominal size) are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\tilde{\alpha}}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\tilde{\alpha}}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 6: Empirical sizes of  $t_{\tilde{\alpha}}^{hac}$  (for  $H_0 : \alpha = 0$ ) in the two-variable LRM:  
HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	10.99% (0.000)***	9.25% (0.000)***	7.93% (0.000)***	6.57% (0.000)***	5.86% (0.000)***	5.49% (0.032)**	4.91% (0.677)	5.22% (0.323)
<b>0.10</b>	12.62% (0.000)***	10.90% (0.000)***	8.88% (0.000)***	7.41% (0.000)***	6.01% (0.000)***	5.97% (0.000)***	5.32% (0.154)	5.48% (0.035)**
<b>0.20</b>	13.85% (0.000)***	11.97% (0.000)***	9.93% (0.000)***	8.08% (0.000)***	6.61% (0.000)***	6.33% (0.000)***	5.78% (0.001)***	5.51% (0.025)**
<b>0.30</b>	15.95% (0.000)***	13.27% (0.000)***	10.69% (0.000)***	8.81% (0.000)***	7.63% (0.000)***	6.95% (0.000)***	6.06% (0.000)***	5.74% (0.002)***
<b>0.40</b>	17.49% (0.000)***	15.76% (0.000)***	12.04% (0.000)***	10.14% (0.000)***	7.86% (0.000)***	7.47% (0.000)***	6.30% (0.000)***	5.95% (0.000)***
<b>0.50</b>	21.10% (0.000)***	16.84% (0.000)***	14.54% (0.000)***	11.93% (0.000)***	8.95% (0.000)***	8.21% (0.000)***	7.43% (0.000)***	6.52% (0.000)***
<b>0.60</b>	24.35% (0.000)***	20.40% (0.000)***	16.61% (0.000)***	13.65% (0.000)***	11.26% (0.000)***	9.64% (0.000)***	8.12% (0.000)***	6.49% (0.000)***
<b>0.70</b>	30.33% (0.000)***	26.21% (0.000)***	20.58% (0.000)***	17.35% (0.000)***	14.64% (0.000)***	12.87% (0.000)***	9.29% (0.000)***	8.47% (0.000)***
<b>0.80</b>	35.23% (0.000)***	32.38% (0.000)***	27.72% (0.000)***	24.65% (0.000)***	20.16% (0.000)***	17.30% (0.000)***	13.15% (0.000)***	11.49% (0.000)***
<b>0.90</b>	43.25% (0.000)***	43.33% (0.000)***	38.76% (0.000)***	38.80% (0.000)***	33.46% (0.000)***	30.16% (0.000)***	22.25% (0.000)***	19.64% (0.000)***
<b>1.00</b>	77.36% (0.000)***	80.71% (0.000)***	82.87% (0.000)***	85.73% (0.000)***	89.50% (0.000)***	90.86% (0.000)***	94.05% (0.000)***	94.82% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The empirical sizes (in percentages) of  $t_{\tilde{\alpha}}^{hac}$  (using a 5% nominal size) are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\tilde{\alpha}}^{hac}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\tilde{\alpha}}^{hac}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 7: Empirical sizes of  $t_{\tilde{\beta}_2}$  (for  $H_0 : \beta_2 = 0$ ) in the two-variable LRM:  
OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.34% (0.131)	4.76% (0.260)	5.11% (0.617)	5.08% (0.716)	5.06% (0.784)	4.86% (0.515)	5.14% (0.526)	5.10% (0.650)
<b>0.10</b>	7.43% (0.000)***	7.34% (0.000)***	7.22% (0.000)***	7.82% (0.000)***	7.43% (0.000)***	7.64% (0.000)***	7.58% (0.000)***	7.81% (0.000)***
<b>0.20</b>	9.01% (0.000)***	10.24% (0.000)***	11.03% (0.000)***	10.65% (0.000)***	10.72% (0.000)***	11.01% (0.000)***	11.28% (0.000)***	11.71% (0.000)***
<b>0.30</b>	12.44% (0.000)***	13.40% (0.000)***	14.46% (0.000)***	14.05% (0.000)***	15.03% (0.000)***	15.13% (0.000)***	14.57% (0.000)***	14.32% (0.000)***
<b>0.40</b>	15.68% (0.000)***	17.88% (0.000)***	18.93% (0.000)***	20.10% (0.000)***	20.14% (0.000)***	19.46% (0.000)***	19.67% (0.000)***	20.23% (0.000)***
<b>0.50</b>	19.40% (0.000)***	22.23% (0.000)***	24.21% (0.000)***	25.77% (0.000)***	25.15% (0.000)***	25.02% (0.000)***	25.90% (0.000)***	25.83% (0.000)***
<b>0.60</b>	24.76% (0.000)***	28.46% (0.000)***	30.72% (0.000)***	32.03% (0.000)***	31.75% (0.000)***	33.22% (0.000)***	32.71% (0.000)***	32.49% (0.000)***
<b>0.70</b>	31.35% (0.000)***	36.05% (0.000)***	38.18% (0.000)***	40.15% (0.000)***	40.31% (0.000)***	40.03% (0.000)***	40.54% (0.000)***	40.69% (0.000)***
<b>0.80</b>	38.59% (0.000)***	44.50% (0.000)***	48.07% (0.000)***	50.15% (0.000)***	50.51% (0.000)***	50.50% (0.000)***	50.63% (0.000)***	51.28% (0.000)***
<b>0.90</b>	44.89% (0.000)***	55.71% (0.000)***	59.25% (0.000)***	63.19% (0.000)***	64.07% (0.000)***	65.06% (0.000)***	65.40% (0.000)***	65.84% (0.000)***
<b>1.00</b>	52.79% (0.000)***	66.53% (0.000)***	76.37% (0.000)***	83.56% (0.000)***	89.85% (0.000)***	92.31% (0.000)***	96.64% (0.000)***	98.08% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The empirical sizes (in percentages) of  $t_{\tilde{\beta}_2}$  (using a 5% nominal size) are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\tilde{\beta}_2}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\tilde{\beta}_2}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 8: Empirical sizes of  $t_{\tilde{\beta}_2}^{hac}$  (for  $H_0 : \beta_2 = 0$ ) in the two-variable LRM:  
HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	10.92% (0.000)***	9.18% (0.000)***	7.99% (0.000)***	6.75% (0.000)***	6.14% (0.000)***	5.44% (0.052)*	5.27% (0.227)	5.65% (0.005)***
<b>0.10</b>	12.35% (0.000)***	10.97% (0.000)***	9.28% (0.000)***	7.44% (0.000)***	6.08% (0.000)***	5.89% (0.000)***	5.42% (0.064)*	5.31% (0.167)
<b>0.20</b>	13.72% (0.000)***	11.94% (0.000)***	9.80% (0.000)***	7.86% (0.000)***	6.63% (0.000)***	6.55% (0.000)***	5.91% (0.000)***	5.72% (0.002)***
<b>0.30</b>	15.88% (0.000)***	13.39% (0.000)***	10.88% (0.000)***	9.10% (0.000)***	7.71% (0.000)***	6.86% (0.000)***	6.31% (0.000)***	5.67% (0.004)***
<b>0.40</b>	17.51% (0.000)***	15.94% (0.000)***	12.24% (0.000)***	10.12% (0.000)***	8.32% (0.000)***	7.61% (0.000)***	6.28% (0.000)***	6.01% (0.000)***
<b>0.50</b>	20.83% (0.000)***	16.65% (0.000)***	13.98% (0.000)***	12.43% (0.000)***	9.01% (0.000)***	8.86% (0.000)***	6.95% (0.000)***	7.02% (0.000)***
<b>0.60</b>	24.11% (0.000)***	20.61% (0.000)***	16.29% (0.000)***	13.77% (0.000)***	11.56% (0.000)***	9.98% (0.000)***	7.78% (0.000)***	7.47% (0.000)***
<b>0.70</b>	30.09% (0.000)***	26.05% (0.000)***	20.40% (0.000)***	17.69% (0.000)***	15.07% (0.000)***	12.79% (0.000)***	9.34% (0.000)***	8.53% (0.000)***
<b>0.80</b>	33.83% (0.000)***	31.70% (0.000)***	27.99% (0.000)***	24.44% (0.000)***	20.78% (0.000)***	17.99% (0.000)***	13.15% (0.000)***	11.43% (0.000)***
<b>0.90</b>	40.02% (0.000)***	42.23% (0.000)***	38.22% (0.000)***	37.78% (0.000)***	33.83% (0.000)***	29.53% (0.000)***	22.94% (0.000)***	19.57% (0.000)***
<b>1.00</b>	47.81% (0.000)***	54.45% (0.000)***	60.73% (0.000)***	68.11% (0.000)***	77.02% (0.000)***	82.09% (0.000)***	90.27% (0.000)***	92.17% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The empirical sizes (in percentages) of  $t_{\tilde{\beta}_2}^{hac}$  (using a 5% nominal size) are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\tilde{\beta}_2}^{hac}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\tilde{\beta}_2}^{hac}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 9: Averages of  $\tilde{R}^2$  in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.10</b>	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.20</b>	0.06 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.30</b>	0.06 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.40</b>	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.50</b>	0.09 (0.000)***	0.05 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.60</b>	0.11 (0.000)***	0.06 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.70</b>	0.13 (0.000)***	0.08 (0.000)***	0.05 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.80</b>	0.16 (0.000)***	0.11 (0.000)***	0.07 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.90</b>	0.19 (0.000)***	0.16 (0.000)***	0.11 (0.000)***	0.07 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>1.00</b>	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The averages of  $\tilde{R}^2$  are reported

P-values of t-tests for the averages of  $\tilde{R}^2$  equalling zero are given in parentheses

\*, \*\* and \*\*\* denote the averages of  $\tilde{R}^2$  are significantly different from zero at the 10%, 5% and 1% levels, respectively

**Table 10: Averages of  $\tilde{s}$  in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →								$\sigma_u \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	0.989 (0.000)***	0.995 (0.000)***	0.997 (0.000)***	0.998 (0.003)***	1.000 (0.911)	1.000 (0.082)*	1.000 (0.904)	1.000 (0.346)	<i>1.000</i>
<b>0.10</b>	0.985 (0.000)***	0.996 (0.000)***	0.999 (0.000)***	1.002 (0.000)***	1.004 (0.014)**	1.005 (0.097)*	1.005 (0.020)**	1.005 (0.133)	<i>1.005</i>
<b>0.20</b>	0.990 (0.000)***	1.006 (0.000)***	1.012 (0.000)***	1.016 (0.000)***	1.019 (0.000)***	1.020 (0.000)***	1.021 (0.552)	1.021 (0.697)	<i>1.021</i>
<b>0.30</b>	1.004 (0.000)***	1.026 (0.000)***	1.035 (0.000)***	1.042 (0.000)***	1.046 (0.000)***	1.047 (0.000)***	1.048 (0.017)**	1.048 (0.920)	<i>1.048</i>
<b>0.40</b>	1.025 (0.000)***	1.057 (0.000)***	1.071 (0.000)***	1.083 (0.000)***	1.088 (0.000)***	1.089 (0.000)***	1.091 (0.014)**	1.091 (0.146)	<i>1.091</i>
<b>0.50</b>	1.064 (0.000)***	1.104 (0.000)***	1.128 (0.000)***	1.142 (0.000)***	1.149 (0.000)***	1.152 (0.000)***	1.154 (0.000)***	1.154 (0.002)***	<i>1.155</i>
<b>0.60</b>	1.109 (0.000)***	1.172 (0.000)***	1.209 (0.000)***	1.227 (0.000)***	1.241 (0.000)***	1.246 (0.000)***	1.249 (0.000)***	1.250 (0.007)***	<i>1.250</i>
<b>0.70</b>	1.178 (0.000)***	1.272 (0.000)***	1.333 (0.000)***	1.364 (0.000)***	1.386 (0.000)***	1.393 (0.000)***	1.399 (0.000)***	1.399 (0.000)***	<i>1.400</i>
<b>0.80</b>	1.275 (0.000)***	1.434 (0.000)***	1.532 (0.000)***	1.598 (0.000)***	1.638 (0.000)***	1.653 (0.000)***	1.663 (0.000)***	1.665 (0.000)***	<i>1.667</i>
<b>0.90</b>	1.426 (0.000)***	1.700 (0.000)***	1.936 (0.000)***	2.095 (0.000)***	2.207 (0.000)***	2.248 (0.000)***	2.285 (0.000)***	2.289 (0.000)***	<i>2.294</i>
<b>1.00</b>	1.652 (0.000)***	2.294 (0.000)***	3.209 (0.000)***	4.510 (0.000)***	7.087 (0.000)***	10.099 (0.000)***	22.361 (0.000)***	31.599 (0.000)***	$\sigma_u$
	<i>5.000</i>	<i>7.071</i>	<i>10.000</i>	<i>14.142</i>	<i>22.361</i>	<i>31.623</i>	<i>70.711</i>	<i>100.000</i>	← ( $\rho = 1$ )

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The averages of  $\tilde{s}$  are reported

P-values of t-tests for the averages of  $\tilde{s}$  equalling their population values ( $\sigma_u$ ) are given in parentheses. The population values of

each  $\rho$  (when  $\rho < 1$ ) are given in italics in the far right column and are calculated with  $\sigma_u = +\sqrt{\frac{1}{1-\rho^2}}$ . The population values for

each value of T when  $\rho = 1$  are given in italics in the bottom row and are calculated with  $\sigma_u = +\sqrt{T}$ .

\*, \*\* and \*\*\* denote the averages of  $\tilde{s}$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 11: Averages of  $\tilde{d}$  in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →								<i>DW</i> ↓
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	2.072 (0.000)***	2.039 (0.000)***	2.018 (0.000)***	2.011 (0.000)***	2.005 (0.000)***	2.002 (0.005)***	2.000 (0.268)	2.000 (0.956)	<i>2.000</i>
<b>0.10</b>	1.903 (0.000)***	1.855 (0.000)***	1.831 (0.000)***	1.816 (0.000)***	1.807 (0.000)***	1.804 (0.000)***	1.801 (0.001)	1.801 (0.002)***	<i>1.800</i>
<b>0.20</b>	1.746 (0.000)***	1.675 (0.000)***	1.640 (0.000)***	1.621 (0.000)***	1.609 (0.000)***	1.604 (0.000)***	1.600 (0.115)	1.600 (0.152)	<i>1.600</i>
<b>0.30</b>	1.593 (0.000)***	1.497 (0.000)***	1.450 (0.000)***	1.425 (0.000)***	1.411 (0.000)***	1.405 (0.000)***	1.401 (0.002)***	1.400 (0.097)*	<i>1.400</i>
<b>0.40</b>	1.431 (0.000)***	1.317 (0.000)***	1.263 (0.000)***	1.228 (0.000)***	1.212 (0.000)***	1.205 (0.000)***	1.201 (0.000)***	1.201 (0.001)***	<i>1.200</i>
<b>0.50</b>	1.272 (0.000)***	1.134 (0.000)***	1.071 (0.000)***	1.036 (0.000)***	1.014 (0.000)***	1.007 (0.000)***	1.002 (0.000)***	1.001 (0.001)***	<i>1.000</i>
<b>0.60</b>	1.124 (0.000)***	0.965 (0.000)***	0.881 (0.000)***	0.844 (0.000)***	0.817 (0.000)***	0.808 (0.000)***	0.801 (0.000)***	0.801 (0.000)***	<i>0.800</i>
<b>0.70</b>	0.976 (0.000)***	0.786 (0.000)***	0.692 (0.000)***	0.646 (0.000)***	0.618 (0.000)***	0.608 (0.000)***	0.602 (0.000)***	0.601 (0.000)***	<i>0.600</i>
<b>0.80</b>	0.839 (0.000)***	0.618 (0.000)***	0.509 (0.000)***	0.451 (0.000)***	0.420 (0.000)***	0.410 (0.000)***	0.402 (0.000)***	0.401 (0.000)***	<i>0.400</i>
<b>0.90</b>	0.706 (0.000)***	0.462 (0.000)***	0.324 (0.000)***	0.259 (0.000)***	0.223 (0.000)***	0.211 (0.000)***	0.202 (0.000)***	0.201 (0.000)***	<i>0.200</i>
<b>1.00</b>	0.612 (0.000)***	0.332 (0.000)***	0.174 (0.000)***	0.089 (0.000)***	0.036 (0.000)***	0.018 (0.000)***	0.004 (0.000)***	0.002 (0.000)***	<i>0.000</i>

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The averages of  $\tilde{d}$  are reported

P-values of t-tests for the averages of  $\tilde{d}$  equalling their population values (*DW*) are given in parentheses. The population value for each value of  $\rho$  (for all T) are given in italics in the far right column and are calculated with  $DW \cong 2(1 - \rho)$ .

\*, \*\* and \*\*\* denote the averages of  $\tilde{d}$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 12: Empirical sizes of  $\tilde{d}$  (for  $H_0: \tilde{d} = 2$ ) in the two-variable LRM**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	2.04% (0.000)***	2.59% (0.000)***	3.08% (0.000)***	3.60% (0.000)***	0.30% (0.000)***	0.00% (0.000)***	0.00% (0.000)***	0.00% (0.000)***
<b>0.10</b>	5.27% (0.227)	10.39% (0.000)***	18.29% (0.000)***	34.22% (0.000)***	29.11% (0.000)***	23.12% (0.000)***	6.51% (0.000)***	1.33% (0.000)***
<b>0.20</b>	11.47% (0.000)***	26.95% (0.000)***	53.75% (0.000)***	83.89% (0.000)***	95.41% (0.000)***	99.27% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>0.30</b>	22.58% (0.000)***	51.85% (0.000)***	85.27% (0.000)***	99.27% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>0.40</b>	37.40% (0.000)***	76.15% (0.000)***	97.73% (0.000)***	99.98% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>0.50</b>	54.59% (0.000)***	91.36% (0.000)***	99.87% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>0.60</b>	69.87% (0.000)***	97.46% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>0.70</b>	81.44% (0.000)***	99.45% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>0.80</b>	88.72% (0.000)***	99.86% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>0.90</b>	93.29% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***
<b>1.00</b>	95.71% (0.000)***	99.99% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***	100.00% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \tilde{\alpha} + \tilde{\beta}_2 X_{2t} + \tilde{u}_t$

The empirical sizes (in percentages) of  $\tilde{d}$  (using a 5% nominal size) are reported. The lower bounds of the DW-distribution are used for critical values.

P-values of t-tests for the empirical size of  $\tilde{d}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $\tilde{d}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

<b>Table 13: Difference of means and chi-square tests for various combinations of <math>\phi</math></b>									
	<b>Including <math>\rho = 1</math> and <math>\phi = 1</math></b>			<b>Excluding <math>\rho = 1</math> and <math>\phi = 1</math></b>			<b>Excluding <math>\rho = 1, \rho = 0.9, \phi = 1</math> and <math>\phi = 0.9</math></b>		
	<b>Empirical size</b>	$\chi^2(54)$	<b>p-value</b>	<b>Empirical size</b>	$\chi^2(44)$	<b>p-value</b>	<b>Empirical size</b>	$\chi^2(35)$	<b>p-value</b>
<b>Coefficient magnitudes</b>									
Intercept, $\hat{\alpha}$	29.65%	17402.68	0.00%	14.86%	1617.25	0.00%	9.76%	338.38	0.00%
$I(0)$ regressor, $\hat{\beta}_1$	89.79%	70519.55	0.00%	86.78%	48989.50	0.00%	80.40%	32422.66	0.00%
$I(1)$ regressor, $\hat{\beta}_2$	31.67%	17419.41	0.00%	17.22%	2266.00	0.00%	12.11%	753.21	0.00%
<b>T-ratios' empirical size</b>									
Intercept, $t_{\hat{\alpha}}$	10.35%	941.14	0.00%	7.08%	133.75	0.00%	4.82%	47.66	6.01%
$I(0)$ regressor, $t_{\hat{\beta}_1}$	85.58%	63368.45	0.00%	83.61%	44976.50	0.00%	77.93%	30138.38	0.00%
$I(1)$ regressor, $t_{\hat{\beta}_2}$	9.50%	528.36	0.00%	6.50%	86.00	0.02%	4.63%	36.66	34.66%
<b>Averages of fit measures</b>									
$R^2$	77.13%	53174.41	0.00%	77.22%	39884.00	0.00%	72.53%	26892.16	0.00%
$s$	35.76%	11785.68	0.00%	31.17%	6410.50	0.00%	23.61%	3028.27	0.00%
<b>Durbin-Watson statistic properties</b>									
Averages of $d$	34.73%	28669.32	0.00%	37.67%	23163.50	0.00%	38.73%	18289.60	0.00%
Empirical sizes of $d$	20.12%	3073.09	0.00%	19.61%	2188.00	0.00%	18.79%	1603.27	0.00%
DGP $Y_{0t}$ : $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ , $Y_0 = 0$ , $e_{0t} \sim N(0, 1)$ ; DGP $X_{1t}$ : $X_{1t} = \phi X_{1,t-1} + e_{1t}$ , $X_{1,0} = 0$ , $e_{1t} \sim N(0, 1)$ ; DGP $X_{2t}$ : $X_{2t} = X_{2,t-1} + v_{2t}$ , $X_{2,0} = 0$ , $v_{2t} \sim N(0, 1)$									
Regression: $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$									
Tests are based on the difference between the means of a statistic being zero at the 5% level. Tests are grouped to ensure sample sizes and values of $\rho$ are the same.									
Tests compare means for different values of $\phi$ using all combinations of $\phi = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$ except when otherwise stated.									
A maximum of 55 samples (for example, the difference in means for $\phi = 0.0$ and $\phi = 0.1$ is one sample) containing 88 elements, equal to the number of sample sizes (8) multiplied by the number of values of $\rho$ (11), are used to apply the chi-square test that the proportion of rejections is not significantly different from 5%. Hence, the tests use up to 54 dof.									
The column headed "Empirical size" gives the percentage of times the null is rejected for the difference in means test. The column headed " $\chi^2(v)$ " is the chi-square test for the percentage of rejections equalling 5%, where $v$ denote the degrees of freedom (dof). The column headed "p-value" gives the probability value of the chi-square test.									

**Table 14a: Magnitudes of  $\hat{\alpha}$  in the three-variable LRM ( $\phi = 0.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	1.86	0.90	0.46	0.24	0.11	0.06	0.02	0.01
<b>0.10</b>	2.03	0.99	0.51	0.27	0.12	0.07	0.02	0.02
<b>0.20</b>	2.16	1.08	0.57	0.30	0.13	0.08	0.03	0.02
<b>0.30</b>	2.34	1.24	0.64	0.34	0.15	0.09	0.03	0.02
<b>0.40</b>	2.61	1.39	0.74	0.39	0.18	0.10	0.03	0.02
<b>0.50</b>	2.90	1.63	0.88	0.48	0.21	0.12	0.04	0.03
<b>0.60</b>	3.45	1.94	1.09	0.58	0.27	0.16	0.05	0.03
<b>0.70</b>	4.09	2.47	1.36	0.78	0.36	0.21	0.07	0.05
<b>0.80</b>	5.09	3.29	1.99	1.12	0.52	0.31	0.10	0.07
<b>0.90</b>	6.64	5.06	3.29	2.03	1.00	0.61	0.21	0.14
<b>1.00</b>	20.91	20.88	20.98	21.49	22.11	23.84	33.55	42.89

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\alpha}$  are reported for  $\phi = 0.0$

**Table 14b: Magnitudes of  $\hat{\alpha}$  in the three-variable LRM ( $\phi = 0.5$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	1.86	0.93	0.47	0.24	0.11	0.06	0.02	0.01
<b>0.10</b>	2.02	1.02	0.51	0.27	0.12	0.07	0.02	0.02
<b>0.20</b>	2.22	1.11	0.58	0.30	0.13	0.08	0.03	0.02
<b>0.30</b>	2.40	1.22	0.65	0.35	0.15	0.09	0.03	0.02
<b>0.40</b>	2.70	1.45	0.73	0.40	0.17	0.10	0.03	0.02
<b>0.50</b>	2.97	1.64	0.90	0.46	0.21	0.12	0.04	0.03
<b>0.60</b>	3.50	1.94	1.09	0.60	0.27	0.15	0.05	0.03
<b>0.70</b>	4.04	2.37	1.39	0.76	0.35	0.20	0.07	0.05
<b>0.80</b>	5.02	3.35	1.97	1.11	0.51	0.31	0.10	0.07
<b>0.90</b>	6.66	4.89	3.30	2.08	1.01	0.61	0.20	0.14
<b>1.00</b>	20.82	21.00	20.92	21.59	22.32	23.79	33.46	42.76

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\alpha}$  are reported for  $\phi = 0.5$

**Table 14c: Magnitudes of  $\hat{\alpha}$  in the three-variable LRM ( $\phi = 0.9$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	2.06	1.00	0.50	0.25	0.11	0.06	0.02	0.01
<b>0.10</b>	2.15	1.11	0.54	0.28	0.12	0.07	0.02	0.02
<b>0.20</b>	2.37	1.19	0.61	0.31	0.14	0.08	0.03	0.02
<b>0.30</b>	2.51	1.32	0.68	0.35	0.16	0.09	0.03	0.02
<b>0.40</b>	2.85	1.50	0.76	0.41	0.18	0.10	0.03	0.02
<b>0.50</b>	3.21	1.74	0.93	0.48	0.22	0.12	0.04	0.03
<b>0.60</b>	3.60	2.07	1.12	0.61	0.27	0.16	0.05	0.03
<b>0.70</b>	4.23	2.55	1.47	0.77	0.36	0.21	0.07	0.05
<b>0.80</b>	5.04	3.32	2.02	1.12	0.53	0.31	0.11	0.07
<b>0.90</b>	6.21	5.01	3.37	2.09	1.01	0.61	0.21	0.14
<b>1.00</b>	20.36	20.81	21.01	21.51	22.58	23.50	33.26	42.67

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\alpha}$  are reported for  $\phi = 0.9$

**Table 14d: Magnitudes of  $\hat{\alpha}$  in the three-variable LRM ( $\phi = 1.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	3.29	1.63	0.84	0.43	0.18	0.10	0.03	0.02
<b>0.10</b>	3.47	1.76	0.93	0.48	0.20	0.11	0.03	0.02
<b>0.20</b>	3.62	1.98	1.00	0.51	0.23	0.13	0.04	0.02
<b>0.30</b>	4.04	2.18	1.13	0.59	0.26	0.14	0.04	0.03
<b>0.40</b>	4.45	2.44	1.30	0.68	0.30	0.16	0.05	0.03
<b>0.50</b>	4.94	2.86	1.50	0.82	0.35	0.20	0.06	0.04
<b>0.60</b>	5.65	3.28	1.84	0.99	0.44	0.24	0.07	0.05
<b>0.70</b>	6.51	4.02	2.35	1.31	0.59	0.32	0.10	0.06
<b>0.80</b>	7.76	5.20	3.17	1.84	0.86	0.48	0.15	0.09
<b>0.90</b>	9.79	7.49	5.22	3.27	1.59	0.94	0.29	0.19
<b>1.00</b>	22.39	22.09	22.21	22.73	23.20	24.79	31.53	38.67

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\alpha}$  are reported for  $\phi = 1.0$

**Table 15a: Magnitudes of  $\hat{\beta}_1$  in the three-variable LRM ( $\phi = 0.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.17	0.12	0.08	0.06	0.04	0.03	0.01	0.01
<b>0.10</b>	0.17	0.12	0.08	0.06	0.04	0.03	0.01	0.01
<b>0.20</b>	0.17	0.12	0.08	0.06	0.04	0.03	0.01	0.01
<b>0.30</b>	0.17	0.12	0.09	0.06	0.04	0.03	0.01	0.01
<b>0.40</b>	0.18	0.12	0.09	0.06	0.04	0.03	0.01	0.01
<b>0.50</b>	0.18	0.13	0.09	0.07	0.04	0.03	0.01	0.01
<b>0.60</b>	0.19	0.14	0.10	0.07	0.04	0.03	0.01	0.01
<b>0.70</b>	0.20	0.15	0.11	0.08	0.05	0.04	0.02	0.01
<b>0.80</b>	0.22	0.17	0.12	0.09	0.06	0.04	0.02	0.01
<b>0.90</b>	0.25	0.20	0.16	0.12	0.08	0.06	0.03	0.02
<b>1.00</b>	0.29	0.27	0.26	0.26	0.25	0.26	0.25	0.25

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_1$  are reported for  $\phi = 0.0$

**Table 15b: Magnitudes of  $\hat{\beta}_1$  in the three-variable LRM ( $\phi = 0.5$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.16	0.11	0.07	0.05	0.03	0.02	0.01	0.01
<b>0.10</b>	0.17	0.11	0.08	0.05	0.03	0.02	0.01	0.01
<b>0.20</b>	0.18	0.12	0.08	0.06	0.03	0.02	0.01	0.01
<b>0.30</b>	0.19	0.12	0.09	0.06	0.04	0.03	0.01	0.01
<b>0.40</b>	0.20	0.14	0.09	0.07	0.04	0.03	0.01	0.01
<b>0.50</b>	0.21	0.15	0.10	0.07	0.05	0.03	0.01	0.01
<b>0.60</b>	0.23	0.16	0.12	0.08	0.05	0.04	0.02	0.01
<b>0.70</b>	0.25	0.19	0.14	0.10	0.06	0.04	0.02	0.01
<b>0.80</b>	0.29	0.22	0.17	0.12	0.08	0.06	0.03	0.02
<b>0.90</b>	0.33	0.28	0.22	0.17	0.11	0.08	0.04	0.03
<b>1.00</b>	0.39	0.38	0.38	0.38	0.38	0.38	0.37	0.38

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_1$  are reported for  $\phi = 0.5$

**Table 15c: Magnitudes of  $\hat{\beta}_1$  in the three-variable LRM ( $\phi = 0.9$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.13	0.07	0.04	0.03	0.02	0.01	0.00	0.00
<b>0.10</b>	0.13	0.08	0.05	0.03	0.02	0.01	0.01	0.00
<b>0.20</b>	0.15	0.08	0.05	0.03	0.02	0.01	0.01	0.00
<b>0.30</b>	0.16	0.09	0.06	0.04	0.02	0.02	0.01	0.00
<b>0.40</b>	0.18	0.11	0.07	0.04	0.03	0.02	0.01	0.01
<b>0.50</b>	0.19	0.12	0.08	0.05	0.03	0.02	0.01	0.01
<b>0.60</b>	0.21	0.14	0.09	0.06	0.04	0.03	0.01	0.01
<b>0.70</b>	0.25	0.17	0.11	0.08	0.05	0.03	0.01	0.01
<b>0.80</b>	0.29	0.22	0.15	0.11	0.07	0.05	0.02	0.01
<b>0.90</b>	0.35	0.29	0.22	0.17	0.11	0.08	0.03	0.02
<b>1.00</b>	0.44	0.44	0.46	0.47	0.47	0.47	0.48	0.47

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_1$  are reported for  $\phi = 0.9$

**Table 15d: Magnitudes of  $\hat{\beta}_1$  in the three-variable LRM ( $\phi = 1.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.12	0.06	0.03	0.01	0.01	0.00	0.00	0.00
<b>0.10</b>	0.12	0.06	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.20</b>	0.13	0.07	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.30</b>	0.14	0.07	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.40</b>	0.16	0.08	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.50</b>	0.18	0.10	0.05	0.03	0.01	0.01	0.00	0.00
<b>0.60</b>	0.20	0.12	0.06	0.03	0.01	0.01	0.00	0.00
<b>0.70</b>	0.23	0.14	0.08	0.04	0.02	0.01	0.00	0.00
<b>0.80</b>	0.28	0.18	0.11	0.06	0.03	0.01	0.00	0.00
<b>0.90</b>	0.34	0.26	0.18	0.11	0.05	0.03	0.01	0.00
<b>1.00</b>	0.43	0.43	0.42	0.43	0.43	0.42	0.43	0.43

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_1$  are reported for  $\phi = 1.0$

**Table 16a: Magnitudes of  $\hat{\beta}_2$  in the three-variable LRM ( $\phi = 0.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.10	0.05	0.02	0.01	0.00	0.00	0.00	0.00
<b>0.10</b>	0.11	0.05	0.03	0.01	0.01	0.00	0.00	0.00
<b>0.20</b>	0.12	0.06	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.30</b>	0.13	0.07	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.40</b>	0.14	0.08	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.50</b>	0.16	0.09	0.05	0.02	0.01	0.00	0.00	0.00
<b>0.60</b>	0.19	0.10	0.06	0.03	0.01	0.01	0.00	0.00
<b>0.70</b>	0.22	0.13	0.07	0.04	0.02	0.01	0.00	0.00
<b>0.80</b>	0.28	0.18	0.10	0.06	0.02	0.01	0.00	0.00
<b>0.90</b>	0.36	0.27	0.17	0.10	0.04	0.02	0.00	0.00
<b>1.00</b>	0.49	0.48	0.49	0.49	0.49	0.49	0.49	0.49

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_2$  are reported for  $\phi = 0.0$

**Table 16b: Magnitudes of  $\hat{\beta}_2$  in the three-variable LRM ( $\phi = 0.5$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.10	0.05	0.03	0.01	0.00	0.00	0.00	0.00
<b>0.10</b>	0.11	0.05	0.03	0.01	0.01	0.00	0.00	0.00
<b>0.20</b>	0.12	0.06	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.30</b>	0.13	0.07	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.40</b>	0.15	0.08	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.50</b>	0.16	0.09	0.05	0.02	0.01	0.00	0.00	0.00
<b>0.60</b>	0.19	0.11	0.06	0.03	0.01	0.01	0.00	0.00
<b>0.70</b>	0.22	0.13	0.07	0.04	0.02	0.01	0.00	0.00
<b>0.80</b>	0.27	0.18	0.10	0.05	0.02	0.01	0.00	0.00
<b>0.90</b>	0.35	0.26	0.17	0.10	0.04	0.02	0.00	0.00
<b>1.00</b>	0.48	0.48	0.48	0.48	0.49	0.48	0.49	0.48

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_2$  are reported for  $\phi = 0.5$

**Table 16c: Magnitudes of  $\hat{\beta}_2$  in the three-variable LRM ( $\phi = 0.9$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.11	0.05	0.03	0.01	0.00	0.00	0.00	0.00
<b>0.10</b>	0.12	0.06	0.03	0.01	0.01	0.00	0.00	0.00
<b>0.20</b>	0.13	0.06	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.30</b>	0.14	0.07	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.40</b>	0.15	0.08	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.50</b>	0.17	0.09	0.05	0.02	0.01	0.00	0.00	0.00
<b>0.60</b>	0.20	0.11	0.06	0.03	0.01	0.01	0.00	0.00
<b>0.70</b>	0.23	0.14	0.08	0.04	0.02	0.01	0.00	0.00
<b>0.80</b>	0.27	0.18	0.10	0.06	0.02	0.01	0.00	0.00
<b>0.90</b>	0.34	0.26	0.17	0.10	0.05	0.02	0.00	0.00
<b>1.00</b>	0.44	0.46	0.47	0.49	0.49	0.47	0.48	0.48

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_2$  are reported for  $\phi = 0.9$

**Table 16d: Magnitudes of  $\hat{\beta}_2$  in the three-variable LRM ( $\phi = 1.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.12	0.06	0.03	0.01	0.01	0.00	0.00	0.00
<b>0.10</b>	0.12	0.06	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.20</b>	0.13	0.07	0.03	0.02	0.01	0.00	0.00	0.00
<b>0.30</b>	0.14	0.08	0.04	0.02	0.01	0.00	0.00	0.00
<b>0.40</b>	0.16	0.09	0.05	0.02	0.01	0.00	0.00	0.00
<b>0.50</b>	0.18	0.10	0.05	0.03	0.01	0.01	0.00	0.00
<b>0.60</b>	0.20	0.12	0.06	0.03	0.01	0.01	0.00	0.00
<b>0.70</b>	0.23	0.14	0.08	0.04	0.02	0.01	0.00	0.00
<b>0.80</b>	0.27	0.18	0.11	0.06	0.03	0.01	0.00	0.00
<b>0.90</b>	0.34	0.26	0.18	0.11	0.05	0.03	0.01	0.00
<b>1.00</b>	0.43	0.42	0.43	0.42	0.43	0.43	0.43	0.43

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The magnitudes of  $\hat{\beta}_2$  are reported for  $\phi = 1.0$

**Table 17a: Empirical sizes of  $t_{\hat{\alpha}}$  (for  $H_0 : \alpha = 0$ ) in the three-variable LRM ( $\phi = 0.5$ ):  
OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.12% (0.586)	5.12% (0.586)	4.83% (0.428)	4.95% (0.818)	4.94% (0.782)	4.92% (0.712)	5.32% (0.154)	5.10% (0.650)
<b>0.10</b>	6.87% (0.000)***	6.95% (0.000)***	7.18% (0.000)***	7.34% (0.000)***	7.49% (0.000)***	7.99% (0.000)***	7.57% (0.000)***	7.92% (0.000)***
<b>0.20</b>	9.46% (0.000)***	10.47% (0.000)***	10.83% (0.000)***	10.46% (0.000)***	10.74% (0.000)***	11.05% (0.000)***	11.61% (0.000)***	10.67% (0.000)***
<b>0.30</b>	12.01% (0.000)***	13.42% (0.000)***	14.97% (0.000)***	14.66% (0.000)***	14.55% (0.000)***	14.68% (0.000)***	14.98% (0.000)***	14.93% (0.000)***
<b>0.40</b>	15.91% (0.000)***	18.72% (0.000)***	18.67% (0.000)***	19.04% (0.000)***	19.28% (0.000)***	19.66% (0.000)***	19.58% (0.000)***	20.55% (0.000)***
<b>0.50</b>	19.87% (0.000)***	22.65% (0.000)***	25.68% (0.000)***	24.78% (0.000)***	25.03% (0.000)***	25.35% (0.000)***	25.37% (0.000)***	25.49% (0.000)***
<b>0.60</b>	25.73% (0.000)***	28.81% (0.000)***	31.00% (0.000)***	32.35% (0.000)***	33.48% (0.000)***	32.52% (0.000)***	33.04% (0.000)***	32.77% (0.000)***
<b>0.70</b>	30.73% (0.000)***	36.16% (0.000)***	38.46% (0.000)***	40.23% (0.000)***	40.69% (0.000)***	41.12% (0.000)***	41.22% (0.000)***	40.86% (0.000)***
<b>0.80</b>	38.56% (0.000)***	45.96% (0.000)***	47.87% (0.000)***	49.60% (0.000)***	51.20% (0.000)***	51.16% (0.000)***	52.18% (0.000)***	51.88% (0.000)***
<b>0.90</b>	47.31% (0.000)***	55.80% (0.000)***	61.20% (0.000)***	64.23% (0.000)***	64.81% (0.000)***	66.41% (0.000)***	64.70% (0.000)***	65.49% (0.000)***
<b>1.00</b>	79.17% (0.000)***	85.50% (0.000)***	89.87% (0.000)***	92.97% (0.000)***	94.50% (0.000)***	96.21% (0.000)***	97.73% (0.000)***	98.50% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\alpha}}$  (using a 5% nominal size) for  $\phi = 0.5$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\alpha}}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\alpha}}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 17b: Empirical sizes of  $t_{\hat{\alpha}}$  (for  $H_0 : \alpha = 0$ ) in the three-variable LRM ( $\phi = 0.9$ ):  
OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.20%	5.13%	5.09%	4.86%	5.09%	5.10%	5.06%	4.72%
	(0.368)	(0.556)	(0.682)	(0.515)	(0.682)	(0.650)	(0.784)	(0.187)
<b>0.10</b>	6.51%	7.47%	7.09%	7.40%	7.39%	7.61%	7.17%	7.74%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	9.31%	9.70%	10.35%	10.57%	10.62%	11.30%	10.97%	11.06%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	11.39%	13.20%	14.18%	15.08%	14.77%	14.71%	15.15%	14.76%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	15.01%	17.61%	18.34%	18.90%	19.59%	20.72%	20.45%	20.11%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	19.32%	22.97%	24.52%	24.51%	25.60%	25.78%	26.35%	25.09%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	23.87%	28.40%	30.93%	31.69%	31.78%	32.91%	31.92%	32.26%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	28.94%	35.11%	39.89%	39.28%	40.86%	41.60%	41.74%	40.60%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	36.02%	43.88%	47.69%	50.28%	51.44%	51.23%	52.62%	51.64%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	43.56%	54.64%	61.22%	63.15%	63.87%	65.28%	64.75%	64.87%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	78.47%	85.45%	89.65%	92.81%	95.10%	95.39%	97.99%	98.27%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\alpha}}$  (using a 5% nominal size) for  $\phi = 0.9$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\alpha}}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\alpha}}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 17c: Empirical sizes of  $t_{\hat{\alpha}}$  (for  $H_0 : \alpha = 0$ ) in the three-variable LRM ( $\phi = 1.0$ ):  
OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.09%	5.11%	5.15%	5.34%	4.89%	5.14%	4.84%	5.43%
	(0.682)	(0.617)	(0.497)	(0.131)	(0.610)	(0.526)	(0.456)	(0.058)*
<b>0.10</b>	6.67%	7.12%	7.82%	7.54%	7.63%	7.38%	7.65%	7.63%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	8.61%	10.31%	10.20%	10.67%	11.16%	11.41%	11.19%	10.95%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	11.21%	13.37%	13.88%	14.32%	15.21%	15.25%	14.75%	15.45%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	14.28%	17.41%	18.59%	19.02%	20.33%	19.61%	20.06%	19.97%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	18.02%	22.51%	23.17%	25.56%	25.02%	26.16%	25.34%	24.85%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	22.44%	26.96%	30.45%	31.57%	32.27%	33.22%	33.79%	32.43%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	27.82%	33.51%	38.07%	39.92%	40.76%	39.79%	40.60%	40.93%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	33.17%	41.20%	46.36%	48.84%	50.74%	51.44%	50.52%	51.70%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	40.37%	51.42%	58.10%	62.03%	64.00%	65.29%	64.93%	66.25%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	69.83%	78.81%	84.47%	89.68%	92.40%	95.04%	97.39%	98.09%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\alpha}}$  (using a 5% nominal size) for  $\phi = 1.0$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\alpha}}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\alpha}}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 18a: Empirical sizes of  $t_{\hat{\alpha}}$  (for  $H_0 : \alpha = 0$ ) in the three-variable LRM ( $\phi = 0.5$ ):  
HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	11.18% (0.000)***	9.58% (0.000)***	7.91% (0.000)***	6.98% (0.000)***	6.18% (0.000)***	5.44% (0.052)*	5.08% (0.716)	4.99% (0.963)
<b>0.10</b>	12.37% (0.000)***	10.54% (0.000)***	8.98% (0.000)***	7.01% (0.000)***	6.22% (0.000)***	6.10% (0.000)***	5.38% (0.092)*	5.59% (0.010)**
<b>0.20</b>	14.42% (0.000)***	11.38% (0.000)***	9.99% (0.000)***	8.21% (0.000)***	7.23% (0.000)***	6.11% (0.000)***	6.09% (0.000)***	5.65% (0.005)***
<b>0.30</b>	15.70% (0.000)***	13.80% (0.000)***	11.33% (0.000)***	9.19% (0.000)***	7.40% (0.000)***	7.27% (0.000)***	5.67% (0.004)***	5.54% (0.018)**
<b>0.40</b>	17.98% (0.000)***	15.04% (0.000)***	12.69% (0.000)***	9.57% (0.000)***	8.08% (0.000)***	7.56% (0.000)***	6.37% (0.000)***	5.50% (0.028)**
<b>0.50</b>	20.85% (0.000)***	17.98% (0.000)***	14.01% (0.000)***	11.78% (0.000)***	9.57% (0.000)***	8.58% (0.000)***	7.06% (0.000)***	6.11% (0.000)***
<b>0.60</b>	25.22% (0.000)***	20.98% (0.000)***	16.66% (0.000)***	14.89% (0.000)***	11.68% (0.000)***	9.88% (0.000)***	7.96% (0.000)***	7.69% (0.000)***
<b>0.70</b>	29.53% (0.000)***	25.59% (0.000)***	21.15% (0.000)***	17.68% (0.000)***	14.26% (0.000)***	12.24% (0.000)***	9.68% (0.000)***	9.01% (0.000)***
<b>0.80</b>	36.76% (0.000)***	33.51% (0.000)***	27.03% (0.000)***	23.78% (0.000)***	20.13% (0.000)***	17.75% (0.000)***	13.12% (0.000)***	11.67% (0.000)***
<b>0.90</b>	44.62% (0.000)***	43.37% (0.000)***	40.65% (0.000)***	38.98% (0.000)***	34.44% (0.000)***	30.28% (0.000)***	22.82% (0.000)***	19.38% (0.000)***
<b>1.00</b>	77.07% (0.000)***	80.54% (0.000)***	82.20% (0.000)***	86.07% (0.000)***	89.20% (0.000)***	91.02% (0.000)***	94.10% (0.000)***	94.95% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\alpha}}$  (using a 5% nominal size) for  $\phi = 0.5$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\alpha}}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\alpha}}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 18b: Empirical sizes of  $t_{\hat{\alpha}}$  (for  $H_0 : \alpha = 0$ ) in the three-variable LRM ( $\phi = 0.9$ ):  
HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	10.78% (0.000)***	9.43% (0.000)***	7.86% (0.000)***	6.65% (0.000)***	5.76% (0.001)***	5.53% (0.020)**	5.42% (0.064)*	5.28% (0.211)
<b>0.10</b>	12.48% (0.000)***	10.77% (0.000)***	9.41% (0.000)***	7.90% (0.000)***	6.46% (0.000)***	5.73% (0.002)***	5.36% (0.110)	5.65% (0.005)***
<b>0.20</b>	14.78% (0.000)***	12.09% (0.000)***	10.22% (0.000)***	8.42% (0.000)***	6.76% (0.000)***	6.34% (0.000)***	5.66% (0.004)***	4.97% (0.890)
<b>0.30</b>	16.14% (0.000)***	13.62% (0.000)***	11.10% (0.000)***	9.00% (0.000)***	7.95% (0.000)***	6.70% (0.000)***	6.01% (0.000)***	5.70% (0.003)***
<b>0.40</b>	18.09% (0.000)***	15.97% (0.000)***	12.03% (0.000)***	10.48% (0.000)***	8.74% (0.000)***	7.42% (0.000)***	6.62% (0.000)***	5.64% (0.005)***
<b>0.50</b>	21.54% (0.000)***	18.53% (0.000)***	14.42% (0.000)***	12.36% (0.000)***	9.65% (0.000)***	9.03% (0.000)***	7.21% (0.000)***	6.07% (0.000)***
<b>0.60</b>	25.57% (0.000)***	21.70% (0.000)***	17.08% (0.000)***	14.60% (0.000)***	11.59% (0.000)***	10.04% (0.000)***	7.87% (0.000)***	7.17% (0.000)***
<b>0.70</b>	28.77% (0.000)***	26.05% (0.000)***	20.98% (0.000)***	17.98% (0.000)***	14.59% (0.000)***	12.62% (0.000)***	10.07% (0.000)***	8.68% (0.000)***
<b>0.80</b>	34.86% (0.000)***	32.27% (0.000)***	27.45% (0.000)***	24.40% (0.000)***	21.58% (0.000)***	17.13% (0.000)***	12.54% (0.000)***	11.58% (0.000)***
<b>0.90</b>	41.57% (0.000)***	42.60% (0.000)***	40.30% (0.000)***	39.46% (0.000)***	34.06% (0.000)***	30.32% (0.000)***	22.87% (0.000)***	20.43% (0.000)***
<b>1.00</b>	77.15% (0.000)***	79.89% (0.000)***	83.04% (0.000)***	86.26% (0.000)***	89.41% (0.000)***	91.07% (0.000)***	93.84% (0.000)***	95.51% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\alpha}}$  (using a 5% nominal size) for  $\phi = 0.9$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\alpha}}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\alpha}}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 18c: Empirical sizes of  $t_{\hat{\alpha}}$  (for  $H_0 : \alpha = 0$ ) in the three-variable LRM ( $\phi = 1.0$ ):  
HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	11.06% (0.000)***	9.24% (0.000)***	8.54% (0.000)***	6.70% (0.000)***	5.81% (0.001)***	5.76% (0.001)***	5.16% (0.470)	5.11% (0.617)
<b>0.10</b>	12.30% (0.000)***	11.16% (0.000)***	9.43% (0.000)***	7.60% (0.000)***	6.52% (0.000)***	5.83% (0.000)***	5.68% (0.003)***	5.10% (0.650)
<b>0.20</b>	14.32% (0.000)***	11.67% (0.000)***	10.27% (0.000)***	8.30% (0.000)***	7.02% (0.000)***	6.74% (0.000)***	5.30% (0.181)	5.40% (0.077)*
<b>0.30</b>	16.15% (0.000)***	13.71% (0.000)***	11.63% (0.000)***	9.08% (0.000)***	8.14% (0.000)***	7.23% (0.000)***	6.30% (0.000)***	6.11% (0.000)***
<b>0.40</b>	18.81% (0.000)***	15.92% (0.000)***	12.78% (0.000)***	10.01% (0.000)***	8.72% (0.000)***	7.10% (0.000)***	6.63% (0.000)***	6.07% (0.000)***
<b>0.50</b>	21.07% (0.000)***	18.62% (0.000)***	14.78% (0.000)***	12.63% (0.000)***	9.38% (0.000)***	8.54% (0.000)***	6.82% (0.000)***	6.71% (0.000)***
<b>0.60</b>	24.51% (0.000)***	21.63% (0.000)***	17.71% (0.000)***	14.56% (0.000)***	12.02% (0.000)***	10.30% (0.000)***	7.61% (0.000)***	6.98% (0.000)***
<b>0.70</b>	27.52% (0.000)***	25.68% (0.000)***	21.16% (0.000)***	18.10% (0.000)***	14.74% (0.000)***	13.12% (0.000)***	9.80% (0.000)***	9.21% (0.000)***
<b>0.80</b>	33.54% (0.000)***	31.03% (0.000)***	27.70% (0.000)***	26.08% (0.000)***	20.86% (0.000)***	17.58% (0.000)***	13.23% (0.000)***	11.24% (0.000)***
<b>0.90</b>	39.00% (0.000)***	39.73% (0.000)***	38.16% (0.000)***	36.96% (0.000)***	32.87% (0.000)***	30.82% (0.000)***	22.83% (0.000)***	19.87% (0.000)***
<b>1.00</b>	68.88% (0.000)***	72.08% (0.000)***	75.22% (0.000)***	80.88% (0.000)***	85.57% (0.000)***	87.46% (0.000)***	92.00% (0.000)***	93.68% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\alpha}}$  (using a 5% nominal size) for  $\phi = 1.0$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\alpha}}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\alpha}}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 19a: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 0.0$ ):**  
**OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	4.69% ( 0.143)	5.12% ( 0.586)	5.13% ( 0.556)	4.81% ( 0.375)	4.43% ( 0.006)***	5.05% ( 0.819)	5.05% ( 0.819)	4.79% ( 0.326)
<b>0.10</b>	5.52% ( 0.023)**	5.02% ( 0.927)	4.99% ( 0.963)	4.94% ( 0.782)	4.73% ( 0.204)	4.80% ( 0.350)	4.95% ( 0.818)	4.91% ( 0.677)
<b>0.20</b>	4.77% ( 0.281)	4.79% ( 0.326)	4.88% ( 0.578)	4.76% ( 0.260)	4.76% ( 0.260)	5.19% ( 0.392)	5.27% ( 0.227)	4.85% ( 0.485)
<b>0.30</b>	4.94% ( 0.782)	5.28% ( 0.211)	5.20% ( 0.368)	5.00% ( 1.000)	5.34% ( 0.131)	4.89% ( 0.610)	4.89% ( 0.610)	5.13% ( 0.556)
<b>0.40</b>	5.30% ( 0.181)	4.78% ( 0.303)	5.21% ( 0.345)	4.86% ( 0.515)	4.87% ( 0.546)	4.69% ( 0.143)	5.00% ( 1.000)	4.74% ( 0.221)
<b>0.50</b>	4.64% ( 0.087)*	4.84% ( 0.456)	4.65% ( 0.097)*	4.73% ( 0.204)	4.87% ( 0.546)	5.02% ( 0.927)	5.45% ( 0.048)**	5.11% ( 0.617)
<b>0.60</b>	4.76% ( 0.260)	5.29% ( 0.195)	5.13% ( 0.556)	5.37% ( 0.101)	4.85% ( 0.485)	4.97% ( 0.890)	4.73% ( 0.204)	4.84% ( 0.456)
<b>0.70</b>	4.85% ( 0.485)	4.95% ( 0.818)	4.88% ( 0.578)	5.14% ( 0.526)	5.31% ( 0.167)	5.06% ( 0.784)	4.86% ( 0.515)	5.17% ( 0.443)
<b>0.80</b>	5.12% ( 0.586)	4.90% ( 0.643)	4.39% ( 0.003)***	5.03% ( 0.891)	4.93% ( 0.747)	4.72% ( 0.187)	4.83% ( 0.428)	4.81% ( 0.375)
<b>0.90</b>	5.01% ( 0.963)	4.95% ( 0.818)	4.97% ( 0.890)	5.32% ( 0.154)	5.36% ( 0.110)	5.19% ( 0.392)	4.89% ( 0.610)	4.82% ( 0.401)
<b>1.00</b>	5.14% ( 0.526)	4.95% ( 0.818)	4.97% ( 0.890)	4.98% ( 0.927)	4.91% ( 0.677)	4.98% ( 0.927)	5.00% ( 1.000)	4.70% ( 0.156)

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 0.0$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 19b: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 0.5$ ):**  
**OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.12% (0.586)	4.80% (0.350)	4.51% (0.018)**	4.89% (0.610)	4.78% (0.303)	5.04% (0.855)	5.17% (0.443)	5.01% (0.963)
<b>0.10</b>	6.09% (0.000)***	6.00% (0.000)***	6.18% (0.000)***	6.40% (0.000)***	6.21% (0.000)***	6.17% (0.000)***	6.26% (0.000)***	6.28% (0.000)***
<b>0.20</b>	7.20% (0.000)***	7.14% (0.000)***	7.14% (0.000)***	7.39% (0.000)***	7.50% (0.000)***	7.69% (0.000)***	7.52% (0.000)***	7.41% (0.000)***
<b>0.30</b>	7.72% (0.000)***	8.57% (0.000)***	8.96% (0.000)***	8.87% (0.000)***	8.89% (0.000)***	9.06% (0.000)***	9.01% (0.000)***	9.43% (0.000)***
<b>0.40</b>	9.35% (0.000)***	10.68% (0.000)***	10.27% (0.000)***	10.85% (0.000)***	10.83% (0.000)***	10.15% (0.000)***	10.44% (0.000)***	10.63% (0.000)***
<b>0.50</b>	11.13% (0.000)***	11.53% (0.000)***	12.53% (0.000)***	13.03% (0.000)***	12.31% (0.000)***	12.92% (0.000)***	12.39% (0.000)***	12.58% (0.000)***
<b>0.60</b>	12.31% (0.000)***	13.65% (0.000)***	15.06% (0.000)***	14.25% (0.000)***	15.42% (0.000)***	15.09% (0.000)***	14.64% (0.000)***	15.33% (0.000)***
<b>0.70</b>	13.90% (0.000)***	16.07% (0.000)***	17.09% (0.000)***	16.69% (0.000)***	18.19% (0.000)***	17.65% (0.000)***	17.15% (0.000)***	17.59% (0.000)***
<b>0.80</b>	15.81% (0.000)***	19.01% (0.000)***	19.19% (0.000)***	19.68% (0.000)***	19.90% (0.000)***	20.16% (0.000)***	20.69% (0.000)***	20.18% (0.000)***
<b>0.90</b>	17.22% (0.000)***	20.43% (0.000)***	21.40% (0.000)***	22.38% (0.000)***	23.48% (0.000)***	22.71% (0.000)***	22.21% (0.000)***	23.08% (0.000)***
<b>1.00</b>	18.70% (0.000)***	21.77% (0.000)***	24.20% (0.000)***	25.04% (0.000)***	25.47% (0.000)***	25.12% (0.000)***	25.13% (0.000)***	25.69% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 0.5$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 19c: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 0.9$ ):  
OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.30%	5.22%	5.01%	4.82%	5.71%	4.96%	5.06%	5.33%
	(0.181)	(0.323)	(0.963)	(0.401)	(0.002)***	(0.854)	(0.784)	(0.142)
<b>0.10</b>	6.48%	6.68%	7.50%	7.45%	7.20%	7.61%	7.05%	7.56%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	8.50%	9.19%	9.23%	10.29%	9.82%	9.96%	10.37%	9.89%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	10.89%	12.35%	12.94%	13.31%	14.17%	13.23%	14.08%	13.63%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	14.05%	15.61%	16.86%	17.31%	18.11%	17.92%	17.94%	17.94%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	17.14%	19.29%	21.82%	22.74%	22.26%	22.69%	22.63%	22.62%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	20.35%	25.03%	27.04%	28.08%	27.86%	28.76%	27.50%	28.37%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	25.27%	30.18%	33.56%	34.62%	34.94%	34.56%	35.37%	34.40%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	29.19%	36.66%	40.52%	41.69%	43.44%	42.79%	43.09%	43.69%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	34.27%	44.07%	48.01%	50.82%	52.55%	50.82%	52.34%	52.30%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	40.02%	50.33%	57.44%	61.20%	62.79%	64.39%	64.90%	65.06%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 0.9$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 19d: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 1.0$ ):**  
**OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.21%	4.89%	4.95%	4.99%	4.82%	4.90%	5.07%	5.30%
	(0.345)	(0.610)	(0.818)	(0.963)	(0.401)	(0.643)	(0.750)	(0.181)
<b>0.10</b>	6.64%	7.04%	8.19%	7.71%	7.99%	7.58%	7.44%	7.94%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	8.77%	9.67%	10.30%	10.58%	11.10%	11.66%	10.89%	10.78%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	11.18%	13.18%	13.86%	14.05%	14.63%	15.01%	14.96%	15.45%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	14.79%	16.79%	18.15%	19.08%	20.48%	20.05%	20.00%	19.95%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	18.08%	22.12%	23.46%	24.55%	24.89%	26.06%	26.48%	25.82%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	22.13%	27.10%	30.25%	32.19%	31.78%	32.48%	33.08%	32.69%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	27.19%	33.00%	36.92%	39.27%	40.18%	40.10%	40.62%	40.74%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	32.62%	41.17%	46.17%	48.53%	50.90%	51.53%	51.39%	50.74%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	39.44%	50.84%	58.15%	61.58%	63.76%	64.12%	64.23%	65.69%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	45.42%	61.10%	72.20%	79.93%	87.35%	91.04%	96.21%	97.27%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 1.0$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 20a: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 0.0$ ):**  
**HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	9.45% (0.000)***	7.72% (0.000)***	6.90% (0.000)***	5.52% (0.023)**	5.60% (0.009)***	5.00% (1.000)	5.04% (0.855)	5.37% (0.101)
<b>0.10</b>	9.27% (0.000)***	7.89% (0.000)***	6.71% (0.000)***	5.99% (0.000)***	5.60% (0.009)***	5.48% (0.035)**	4.80% (0.350)	5.05% (0.819)
<b>0.20</b>	9.65% (0.000)***	8.13% (0.000)***	6.95% (0.000)***	5.80% (0.001)***	5.72% (0.002)***	5.43% (0.058)*	5.47% (0.039)**	4.60% (0.056)*
<b>0.30</b>	9.77% (0.000)***	8.02% (0.000)***	6.68% (0.000)***	5.93% (0.000)***	5.84% (0.000)***	5.39% (0.084)*	4.95% (0.818)	5.03% (0.891)
<b>0.40</b>	9.79% (0.000)***	8.33% (0.000)***	6.90% (0.000)***	6.03% (0.000)***	5.68% (0.003)***	5.19% (0.392)	5.06% (0.784)	4.93% (0.747)
<b>0.50</b>	9.77% (0.000)***	8.01% (0.000)***	6.48% (0.000)***	6.74% (0.000)***	5.31% (0.167)	5.34% (0.131)	4.82% (0.401)	5.02% (0.927)
<b>0.60</b>	9.49% (0.000)***	8.47% (0.000)***	6.27% (0.000)***	6.17% (0.000)***	5.64% (0.005)***	5.63% (0.006)***	4.94% (0.782)	5.03% (0.891)
<b>0.70</b>	9.45% (0.000)***	8.17% (0.000)***	7.12% (0.000)***	5.78% (0.001)***	5.38% (0.092)*	5.46% (0.043)**	4.91% (0.677)	5.35% (0.120)
<b>0.80</b>	9.89% (0.000)***	7.95% (0.000)***	6.37% (0.000)***	6.15% (0.000)***	5.19% (0.392)	5.26% (0.244)	5.01% (0.963)	5.03% (0.891)
<b>0.90</b>	9.59% (0.000)***	7.88% (0.000)***	7.07% (0.000)***	5.89% (0.000)***	5.59% (0.010)**	5.25% (0.262)	5.11% (0.617)	5.20% (0.368)
<b>1.00</b>	9.70% (0.000)***	7.82% (0.000)***	7.16% (0.000)***	6.13% (0.000)***	5.18% (0.417)	5.26% (0.244)	5.02% (0.927)	5.19% (0.392)

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 0.0$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 20b: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 0.5$ ):  
HAC coefficient standard errors**

$\rho \downarrow$	Sample size							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	10.31% (0.000)***	8.58% (0.000)***	6.99% (0.000)***	6.15% (0.000)***	5.57% (0.013)**	5.55% (0.016)**	5.09% (0.682)	5.22% (0.323)
<b>0.10</b>	11.60% (0.000)***	9.00% (0.000)***	7.95% (0.000)***	6.41% (0.000)***	5.98% (0.000)***	5.84% (0.000)***	5.39% (0.084)*	5.38% (0.092)*
<b>0.20</b>	11.81% (0.000)***	10.34% (0.000)***	7.93% (0.000)***	7.37% (0.000)***	5.82% (0.001)***	5.71% (0.002)***	5.15% (0.497)	4.89% (0.610)
<b>0.30</b>	12.78% (0.000)***	10.99% (0.000)***	8.50% (0.000)***	7.46% (0.000)***	5.79% (0.001)***	5.99% (0.000)***	5.40% (0.077)*	4.89% (0.610)
<b>0.40</b>	13.60% (0.000)***	11.83% (0.000)***	9.31% (0.000)***	7.80% (0.000)***	6.83% (0.000)***	6.24% (0.000)***	5.41% (0.070)*	5.42% (0.064)*
<b>0.50</b>	15.76% (0.000)***	12.63% (0.000)***	9.75% (0.000)***	8.23% (0.000)***	7.07% (0.000)***	6.80% (0.000)***	6.02% (0.000)***	5.86% (0.000)***
<b>0.60</b>	16.10% (0.000)***	12.75% (0.000)***	10.95% (0.000)***	8.77% (0.000)***	7.26% (0.000)***	6.98% (0.000)***	5.95% (0.000)***	5.80% (0.001)***
<b>0.70</b>	17.51% (0.000)***	14.36% (0.000)***	11.67% (0.000)***	9.52% (0.000)***	7.57% (0.000)***	7.03% (0.000)***	6.04% (0.000)***	5.68% (0.003)***
<b>0.80</b>	18.25% (0.000)***	15.17% (0.000)***	12.21% (0.000)***	10.06% (0.000)***	8.15% (0.000)***	7.92% (0.000)***	6.42% (0.000)***	6.34% (0.000)***
<b>0.90</b>	19.98% (0.000)***	16.60% (0.000)***	12.86% (0.000)***	11.15% (0.000)***	8.71% (0.000)***	8.01% (0.000)***	6.76% (0.000)***	5.97% (0.000)***
<b>1.00</b>	19.78% (0.000)***	17.05% (0.000)***	14.04% (0.000)***	11.76% (0.000)***	9.60% (0.000)***	8.98% (0.000)***	6.68% (0.000)***	6.39% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 0.5$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 20c: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 0.9$ ):  
HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	10.40% (0.000)***	9.30% (0.000)***	8.04% (0.000)***	6.69% (0.000)***	6.07% (0.000)***	5.62% (0.007)***	5.30% (0.181)	5.24% (0.282)
<b>0.10</b>	12.15% (0.000)***	10.54% (0.000)***	9.56% (0.000)***	7.47% (0.000)***	6.47% (0.000)***	6.07% (0.000)***	5.19% (0.392)	5.24% (0.282)
<b>0.20</b>	13.84% (0.000)***	12.10% (0.000)***	9.80% (0.000)***	7.67% (0.000)***	6.64% (0.000)***	6.40% (0.000)***	5.25% (0.262)	5.54% (0.018)**
<b>0.30</b>	15.71% (0.000)***	13.43% (0.000)***	11.06% (0.000)***	8.65% (0.000)***	7.78% (0.000)***	6.58% (0.000)***	5.50% (0.028)**	5.98% (0.000)***
<b>0.40</b>	17.55% (0.000)***	14.67% (0.000)***	12.53% (0.000)***	10.15% (0.000)***	8.03% (0.000)***	7.08% (0.000)***	6.37% (0.000)***	6.24% (0.000)***
<b>0.50</b>	20.69% (0.000)***	17.56% (0.000)***	13.96% (0.000)***	11.31% (0.000)***	9.86% (0.000)***	7.88% (0.000)***	6.68% (0.000)***	6.57% (0.000)***
<b>0.60</b>	22.48% (0.000)***	19.16% (0.000)***	15.36% (0.000)***	13.15% (0.000)***	11.03% (0.000)***	9.14% (0.000)***	7.63% (0.000)***	6.99% (0.000)***
<b>0.70</b>	26.19% (0.000)***	23.16% (0.000)***	18.93% (0.000)***	16.41% (0.000)***	12.90% (0.000)***	10.49% (0.000)***	8.10% (0.000)***	7.64% (0.000)***
<b>0.80</b>	29.54% (0.000)***	26.81% (0.000)***	23.08% (0.000)***	20.06% (0.000)***	16.09% (0.000)***	13.37% (0.000)***	9.72% (0.000)***	9.20% (0.000)***
<b>0.90</b>	33.74% (0.000)***	32.88% (0.000)***	29.96% (0.000)***	27.26% (0.000)***	22.23% (0.000)***	19.15% (0.000)***	13.68% (0.000)***	11.83% (0.000)***
<b>1.00</b>	37.86% (0.000)***	40.01% (0.000)***	37.90% (0.000)***	38.04% (0.000)***	33.04% (0.000)***	29.72% (0.000)***	22.47% (0.000)***	20.26% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 0.9$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 20d: Empirical sizes of  $t_{\hat{\beta}_1}$  (for  $H_0 : \hat{\beta}_1 = 0$ ) in the three-variable LRM ( $\phi = 1.0$ ):  
HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	11.22% (0.000)***	9.77% (0.000)***	8.19% (0.000)***	7.09% (0.000)***	6.19% (0.000)***	5.57% (0.013)**	5.11% (0.617)	4.82% (0.401)
<b>0.10</b>	12.45% (0.000)***	10.92% (0.000)***	9.62% (0.000)***	7.60% (0.000)***	6.76% (0.000)***	5.96% (0.000)***	5.45% (0.048)**	5.34% (0.131)
<b>0.20</b>	14.23% (0.000)***	12.10% (0.000)***	10.46% (0.000)***	8.24% (0.000)***	7.26% (0.000)***	6.50% (0.000)***	5.52% (0.023)**	5.41% (0.070)*
<b>0.30</b>	16.46% (0.000)***	13.68% (0.000)***	11.67% (0.000)***	9.41% (0.000)***	7.26% (0.000)***	6.82% (0.000)***	6.01% (0.000)***	6.07% (0.000)***
<b>0.40</b>	18.33% (0.000)***	16.19% (0.000)***	12.45% (0.000)***	10.16% (0.000)***	8.60% (0.000)***	7.62% (0.000)***	6.15% (0.000)***	6.42% (0.000)***
<b>0.50</b>	20.69% (0.000)***	17.99% (0.000)***	14.16% (0.000)***	12.34% (0.000)***	9.73% (0.000)***	8.47% (0.000)***	7.19% (0.000)***	6.40% (0.000)***
<b>0.60</b>	24.01% (0.000)***	21.32% (0.000)***	17.03% (0.000)***	14.32% (0.000)***	12.28% (0.000)***	10.62% (0.000)***	8.35% (0.000)***	7.29% (0.000)***
<b>0.70</b>	28.20% (0.000)***	25.95% (0.000)***	21.77% (0.000)***	18.73% (0.000)***	14.59% (0.000)***	13.33% (0.000)***	9.77% (0.000)***	8.57% (0.000)***
<b>0.80</b>	32.54% (0.000)***	31.31% (0.000)***	27.37% (0.000)***	25.17% (0.000)***	20.89% (0.000)***	18.07% (0.000)***	13.31% (0.000)***	11.88% (0.000)***
<b>0.90</b>	38.18% (0.000)***	39.42% (0.000)***	37.39% (0.000)***	36.58% (0.000)***	32.83% (0.000)***	30.14% (0.000)***	22.56% (0.000)***	20.02% (0.000)***
<b>1.00</b>	43.73% (0.000)***	49.48% (0.000)***	54.50% (0.000)***	63.73% (0.000)***	74.21% (0.000)***	79.10% (0.000)***	88.43% (0.000)***	90.98% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_1}$  (using a 5% nominal size) for  $\phi = 1.0$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_1}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_1}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 21a: Empirical sizes of  $t_{\hat{\beta}_2}$  (for  $H_0 : \hat{\beta}_2 = 0$ ) in the two-variable LRM ( $\phi = 0.5$ ):**  
**OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.15%	5.19%	4.82%	4.86%	4.70%	4.86%	5.11%	5.08%
	(0.497)	(0.392)	(0.401)	(0.515)	(0.156)	(0.515)	(0.617)	(0.716)
<b>0.10</b>	7.03%	7.02%	7.33%	7.34%	7.63%	7.94%	7.74%	8.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	9.64%	10.08%	10.87%	10.36%	10.44%	10.72%	11.92%	10.73%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	12.09%	13.43%	14.44%	14.68%	14.33%	14.83%	15.09%	14.46%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	15.51%	18.14%	18.54%	18.71%	19.76%	19.79%	19.67%	20.59%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	19.18%	22.13%	24.57%	25.11%	25.24%	25.61%	25.33%	25.59%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	24.62%	28.20%	31.01%	32.76%	32.61%	32.33%	32.42%	33.05%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	29.71%	34.53%	38.57%	39.26%	39.79%	40.97%	40.73%	40.90%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	36.65%	44.60%	47.24%	48.73%	50.49%	51.19%	51.78%	51.43%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	43.19%	53.62%	60.13%	62.13%	63.87%	65.61%	64.64%	65.08%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	50.94%	66.22%	75.89%	83.22%	89.18%	92.43%	96.85%	97.53%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_2}$  (using a 5% nominal size) for  $\phi = 0.5$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_2}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_2}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 21b: Empirical sizes of  $t_{\hat{\beta}_2}$  (for  $H_0 : \hat{\beta}_2 = 0$ ) in the two-variable LRM ( $\phi = 0.9$ ):**  
**OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.46% (0.043)**	5.08% (0.716)	4.99% (0.963)	4.98% (0.927)	5.04% (0.855)	5.10% (0.650)	5.29% (0.195)	4.84% (0.456)
<b>0.10</b>	6.87% (0.000)***	7.40% (0.000)***	7.09% (0.000)***	7.40% (0.000)***	7.69% (0.000)***	7.83% (0.000)***	7.00% (0.000)***	8.20% (0.000)***
<b>0.20</b>	8.89% (0.000)***	9.44% (0.000)***	9.95% (0.000)***	10.39% (0.000)***	10.59% (0.000)***	11.13% (0.000)***	11.24% (0.000)***	11.14% (0.000)***
<b>0.30</b>	11.31% (0.000)***	13.19% (0.000)***	14.03% (0.000)***	15.02% (0.000)***	14.72% (0.000)***	14.91% (0.000)***	15.16% (0.000)***	14.81% (0.000)***
<b>0.40</b>	14.83% (0.000)***	17.32% (0.000)***	18.02% (0.000)***	19.25% (0.000)***	19.13% (0.000)***	20.53% (0.000)***	19.78% (0.000)***	20.05% (0.000)***
<b>0.50</b>	19.20% (0.000)***	22.01% (0.000)***	24.28% (0.000)***	24.64% (0.000)***	25.63% (0.000)***	25.30% (0.000)***	25.77% (0.000)***	26.01% (0.000)***
<b>0.60</b>	22.97% (0.000)***	27.98% (0.000)***	30.24% (0.000)***	31.98% (0.000)***	31.24% (0.000)***	32.32% (0.000)***	33.24% (0.000)***	32.22% (0.000)***
<b>0.70</b>	28.03% (0.000)***	34.44% (0.000)***	38.53% (0.000)***	39.89% (0.000)***	41.10% (0.000)***	41.37% (0.000)***	41.19% (0.000)***	40.82% (0.000)***
<b>0.80</b>	34.29% (0.000)***	42.12% (0.000)***	47.21% (0.000)***	49.18% (0.000)***	49.86% (0.000)***	50.90% (0.000)***	51.20% (0.000)***	51.49% (0.000)***
<b>0.90</b>	40.45% (0.000)***	52.35% (0.000)***	58.52% (0.000)***	62.33% (0.000)***	63.60% (0.000)***	65.28% (0.000)***	64.81% (0.000)***	65.03% (0.000)***
<b>1.00</b>	46.67% (0.000)***	64.19% (0.000)***	75.00% (0.000)***	83.22% (0.000)***	89.29% (0.000)***	92.39% (0.000)***	96.80% (0.000)***	97.66% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_2}$  (using a 5% nominal size) for  $\phi = 0.9$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_2}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_2}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 21c: Empirical sizes of  $t_{\hat{\beta}_2}$  (for  $H_0 : \hat{\beta}_2 = 0$ ) in the two-variable LRM ( $\phi = 1.0$ ):**  
**OLS coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	5.30%	5.29%	5.11%	5.15%	4.69%	5.18%	5.18%	5.25%
	(0.181)	(0.195)	(0.617)	(0.497)	(0.143)	(0.417)	(0.417)	(0.262)
<b>0.10</b>	6.34%	7.13%	7.40%	7.32%	7.63%	7.25%	7.89%	7.45%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	8.79%	9.88%	10.57%	10.26%	10.94%	11.35%	10.68%	10.45%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	11.24%	13.23%	13.82%	14.07%	15.62%	14.80%	14.32%	15.08%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	14.44%	17.23%	18.08%	18.88%	20.05%	19.96%	19.66%	19.49%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	18.17%	21.97%	23.11%	24.22%	24.66%	26.32%	26.17%	25.47%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	22.18%	27.24%	30.10%	31.32%	32.24%	32.83%	33.31%	32.42%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	26.98%	33.54%	37.53%	39.81%	39.77%	40.61%	40.81%	40.82%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	33.11%	40.88%	46.63%	49.39%	50.95%	50.97%	51.47%	51.53%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	38.46%	50.59%	57.12%	61.52%	62.99%	64.53%	65.54%	65.22%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	44.49%	60.11%	71.08%	80.86%	87.13%	91.75%	96.31%	97.32%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_2}$  (using a 5% nominal size) for  $\phi = 1.0$  are reported. OLS coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_2}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_2}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 22a: Empirical sizes of  $t_{\hat{\beta}_2}$  (for  $H_0 : \hat{\beta}_2 = 0$ ) in the two-variable LRM ( $\phi = 0.5$ ):  
HAC coefficient standard errors**

$\rho \downarrow$	Sample size							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	10.98% (0.000)***	9.52% (0.000)***	7.88% (0.000)***	7.01% (0.000)***	6.14% (0.000)***	5.66% (0.004)***	5.26% (0.244)	4.86% (0.515)
<b>0.10</b>	12.63% (0.000)***	10.66% (0.000)***	9.04% (0.000)***	7.40% (0.000)***	6.42% (0.000)***	5.98% (0.000)***	5.87% (0.000)***	5.21% (0.345)
<b>0.20</b>	14.60% (0.000)***	11.32% (0.000)***	10.09% (0.000)***	8.28% (0.000)***	7.00% (0.000)***	6.43% (0.000)***	5.78% (0.001)***	5.21% (0.345)
<b>0.30</b>	15.80% (0.000)***	13.93% (0.000)***	11.44% (0.000)***	9.22% (0.000)***	6.96% (0.000)***	7.24% (0.000)***	5.78% (0.001)***	5.81% (0.001)***
<b>0.40</b>	18.22% (0.000)***	14.75% (0.000)***	12.96% (0.000)***	10.01% (0.000)***	8.19% (0.000)***	7.29% (0.000)***	6.35% (0.000)***	5.80% (0.001)***
<b>0.50</b>	20.95% (0.000)***	17.74% (0.000)***	14.27% (0.000)***	11.54% (0.000)***	9.22% (0.000)***	8.95% (0.000)***	7.03% (0.000)***	6.72% (0.000)***
<b>0.60</b>	24.42% (0.000)***	20.74% (0.000)***	17.13% (0.000)***	15.16% (0.000)***	11.48% (0.000)***	9.73% (0.000)***	8.04% (0.000)***	7.57% (0.000)***
<b>0.70</b>	28.90% (0.000)***	24.72% (0.000)***	21.05% (0.000)***	17.83% (0.000)***	14.17% (0.000)***	12.18% (0.000)***	9.61% (0.000)***	9.07% (0.000)***
<b>0.80</b>	34.49% (0.000)***	31.97% (0.000)***	27.09% (0.000)***	24.52% (0.000)***	20.42% (0.000)***	17.64% (0.000)***	13.32% (0.000)***	11.31% (0.000)***
<b>0.90</b>	41.24% (0.000)***	40.81% (0.000)***	38.99% (0.000)***	37.98% (0.000)***	34.14% (0.000)***	30.65% (0.000)***	22.66% (0.000)***	20.01% (0.000)***
<b>1.00</b>	47.92% (0.000)***	54.42% (0.000)***	61.16% (0.000)***	68.81% (0.000)***	77.46% (0.000)***	81.72% (0.000)***	90.10% (0.000)***	92.77% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_2}$  (using a 5% nominal size) for  $\phi = 0.5$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_2}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_2}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 22b: Empirical sizes of  $t_{\hat{\beta}_2}$  (for  $H_0 : \hat{\beta}_2 = 0$ ) in the two-variable LRM ( $\phi = 0.9$ ):**  
**HAC coefficient standard errors**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	10.69%	9.64%	7.86%	6.71%	6.32%	5.68%	5.35%	5.14%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.003)***	(0.120)	(0.526)
<b>0.10</b>	12.40%	10.68%	9.25%	7.76%	6.16%	5.87%	5.44%	5.49%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.052)*	(0.032)**
<b>0.20</b>	14.50%	12.04%	10.46%	8.74%	7.34%	6.19%	5.69%	5.16%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.003)***	(0.470)
<b>0.30</b>	16.17%	13.78%	11.36%	9.09%	7.64%	6.67%	6.24%	5.97%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	18.25%	15.62%	12.70%	10.72%	8.80%	7.18%	6.39%	6.05%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	21.79%	18.19%	14.50%	12.08%	10.12%	8.83%	7.23%	6.85%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	24.98%	21.40%	17.27%	14.99%	11.78%	9.57%	8.09%	7.68%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	28.66%	25.22%	21.36%	17.54%	14.80%	12.50%	10.18%	8.93%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	33.86%	31.02%	27.64%	24.27%	21.17%	16.70%	13.28%	11.71%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	37.80%	40.03%	38.82%	39.35%	34.34%	30.25%	22.59%	19.41%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	46.21%	53.14%	60.02%	67.97%	77.59%	81.67%	90.72%	92.67%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_2}$  (using a 5% nominal size) for  $\phi = 0.9$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_2}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_2}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 22c: Empirical sizes of  $t_{\hat{\beta}_2}$  (for  $H_0 : \hat{\beta}_2 = 0$ ) in the two-variable LRM ( $\phi = 1.0$ ):**  
**HAC coefficient standard errors**

$\rho \downarrow$	Sample size							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	11.24% (0.000)***	9.55% (0.000)***	8.36% (0.000)***	7.06% (0.000)***	6.04% (0.000)***	5.39% (0.084)*	5.26% (0.244)	5.39% (0.084)*
<b>0.10</b>	12.63% (0.000)***	11.47% (0.000)***	9.54% (0.000)***	7.34% (0.000)***	6.16% (0.000)***	5.70% (0.003)***	5.99% (0.000)***	5.26% (0.244)
<b>0.20</b>	14.04% (0.000)***	12.63% (0.000)***	10.31% (0.000)***	8.21% (0.000)***	7.06% (0.000)***	6.74% (0.000)***	5.26% (0.244)	5.73% (0.002)***
<b>0.30</b>	16.50% (0.000)***	13.87% (0.000)***	10.93% (0.000)***	9.24% (0.000)***	7.92% (0.000)***	7.35% (0.000)***	6.22% (0.000)***	5.77% (0.001)***
<b>0.40</b>	19.07% (0.000)***	15.44% (0.000)***	13.20% (0.000)***	10.02% (0.000)***	8.37% (0.000)***	7.40% (0.000)***	6.30% (0.000)***	5.96% (0.000)***
<b>0.50</b>	20.92% (0.000)***	18.87% (0.000)***	14.33% (0.000)***	12.61% (0.000)***	9.76% (0.000)***	8.74% (0.000)***	7.04% (0.000)***	6.63% (0.000)***
<b>0.60</b>	24.40% (0.000)***	21.39% (0.000)***	17.67% (0.000)***	14.44% (0.000)***	11.73% (0.000)***	10.26% (0.000)***	7.81% (0.000)***	7.05% (0.000)***
<b>0.70</b>	28.30% (0.000)***	25.38% (0.000)***	20.96% (0.000)***	18.17% (0.000)***	14.66% (0.000)***	13.35% (0.000)***	9.87% (0.000)***	8.51% (0.000)***
<b>0.80</b>	33.22% (0.000)***	31.04% (0.000)***	26.33% (0.000)***	25.21% (0.000)***	20.95% (0.000)***	18.14% (0.000)***	13.33% (0.000)***	11.31% (0.000)***
<b>0.90</b>	38.19% (0.000)***	38.63% (0.000)***	36.99% (0.000)***	36.36% (0.000)***	33.73% (0.000)***	30.45% (0.000)***	22.89% (0.000)***	20.12% (0.000)***
<b>1.00</b>	43.18% (0.000)***	48.88% (0.000)***	55.69% (0.000)***	64.18% (0.000)***	73.66% (0.000)***	79.16% (0.000)***	88.33% (0.000)***	90.77% (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_{0,0} = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $t_{\hat{\beta}_2}$  (using a 5% nominal size) for  $\phi = 1.0$  are reported. HAC coefficient standard errors are employed.

P-values of t-tests for the empirical size of  $t_{\hat{\beta}_2}$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $t_{\hat{\beta}_2}$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 23a: Averages of  $R^2$  in the three-variable LRM ( $\phi = 0.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.10</b>	0.09 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.20</b>	0.10 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.30</b>	0.10 (0.000)***	0.06 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.40</b>	0.12 (0.000)***	0.06 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.50</b>	0.13 (0.000)***	0.07 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.60</b>	0.14 (0.000)***	0.08 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.70</b>	0.17 (0.000)***	0.10 (0.000)***	0.06 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.80</b>	0.20 (0.000)***	0.13 (0.000)***	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.90</b>	0.23 (0.000)***	0.18 (0.000)***	0.12 (0.000)***	0.08 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>1.00</b>	0.28 (0.000)***	0.26 (0.000)***	0.25 (0.000)***	0.25 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $R^2$  for  $\phi = 0.0$  are reported

P-values of t-tests for the averages of  $R^2$  equalling zero are given in parentheses

\*, \*\* and \*\*\* denote the averages of  $R^2$  are significantly different from zero at the 10%, 5% and 1% levels, respectively

**Table 23b: Averages of  $R^2$  in the three-variable LRM ( $\phi = 0.5$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.10</b>	0.09 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.20</b>	0.10 (0.000)***	0.05 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.30</b>	0.12 (0.000)***	0.06 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.40</b>	0.13 (0.000)***	0.07 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.50</b>	0.15 (0.000)***	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.60</b>	0.17 (0.000)***	0.10 (0.000)***	0.05 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.70</b>	0.19 (0.000)***	0.12 (0.000)***	0.07 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.80</b>	0.22 (0.000)***	0.15 (0.000)***	0.09 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.90</b>	0.26 (0.000)***	0.19 (0.000)***	0.13 (0.000)***	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>1.00</b>	0.31 (0.000)***	0.28 (0.000)***	0.26 (0.000)***	0.25 (0.000)***	0.25 (0.000)***	0.24 (0.000)***	0.24 (0.000)***	0.24 (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $R^2$  for  $\phi = 0.5$  are reported

P-values of t-tests for the averages of  $R^2$  equalling zero are given in parentheses

\*, \*\* and \*\*\* denote the averages of  $R^2$  are significantly different from zero at the 10%, 5% and 1% levels, respectively

**Table 23c: Averages of  $R^2$  in the three-variable LRM ( $\phi = 0.9$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.10</b>	0.09 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.20</b>	0.11 (0.000)***	0.06 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.30</b>	0.12 (0.000)***	0.07 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.40</b>	0.14 (0.000)***	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.50</b>	0.17 (0.000)***	0.09 (0.000)***	0.05 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.60</b>	0.19 (0.000)***	0.12 (0.000)***	0.07 (0.000)***	0.04 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.70</b>	0.23 (0.000)***	0.14 (0.000)***	0.09 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.80</b>	0.27 (0.000)***	0.19 (0.000)***	0.12 (0.000)***	0.07 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.90</b>	0.32 (0.000)***	0.25 (0.000)***	0.18 (0.000)***	0.11 (0.000)***	0.05 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.00 (0.000)***
<b>1.00</b>	0.38 (0.000)***	0.35 (0.000)***	0.32 (0.000)***	0.29 (0.000)***	0.27 (0.000)***	0.25 (0.000)***	0.24 (0.000)***	0.24 (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_{0,0} = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $R^2$  for  $\phi = 0.9$  are reported

P-values of t-tests for the averages of  $R^2$  equalling zero are given in parentheses

\*, \*\* and \*\*\* denote the averages of  $R^2$  are significantly different from zero at the 10%, 5% and 1% levels, respectively

**Table 23d: Averages of  $R^2$  in the three-variable LRM ( $\phi = 1.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.09 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.10</b>	0.09 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.20</b>	0.11 (0.000)***	0.06 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.30</b>	0.12 (0.000)***	0.07 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.40</b>	0.14 (0.000)***	0.08 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.50</b>	0.17 (0.000)***	0.10 (0.000)***	0.05 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.60</b>	0.20 (0.000)***	0.12 (0.000)***	0.07 (0.000)***	0.04 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.70</b>	0.23 (0.000)***	0.15 (0.000)***	0.09 (0.000)***	0.05 (0.000)***	0.02 (0.000)***	0.01 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.80</b>	0.28 (0.000)***	0.20 (0.000)***	0.13 (0.000)***	0.08 (0.000)***	0.03 (0.000)***	0.02 (0.000)***	0.00 (0.000)***	0.00 (0.000)***
<b>0.90</b>	0.34 (0.000)***	0.27 (0.000)***	0.20 (0.000)***	0.13 (0.000)***	0.06 (0.000)***	0.03 (0.000)***	0.01 (0.000)***	0.00 (0.000)***
<b>1.00</b>	0.40 (0.000)***	0.40 (0.000)***	0.40 (0.000)***	0.40 (0.000)***	0.40 (0.000)***	0.40 (0.000)***	0.40 (0.000)***	0.40 (0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $R^2$  for  $\phi = 1.0$  are reported

P-values of t-tests for the averages of  $R^2$  equalling zero are given in parentheses

\*, \*\* and \*\*\* denote the averages of  $R^2$  are significantly different from zero at the 10%, 5% and 1% levels, respectively

**Table 24a: Averages of  $s$  in the three-variable LRM ( $\phi = 0.0$ )**

$\rho \downarrow$	← Sample size (T) →								$\sigma_u \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	0.989 (0.000)***	0.995 (0.000)***	0.997 (0.000)***	0.999 (0.004)***	1.000 (0.809)	1.000 (0.085)*	1.000 (0.904)	1.000 (0.346)	<i>1.000</i>
<b>0.10</b>	0.984 (0.000)***	0.996 (0.000)***	0.999 (0.000)***	1.002 (0.000)***	1.004 (0.015)**	1.005 (0.109)	1.005 (0.020)**	1.005 (0.129)	<i>1.005</i>
<b>0.20</b>	0.990 (0.000)***	1.006 (0.000)***	1.012 (0.000)***	1.016 (0.000)***	1.019 (0.000)***	1.020 (0.000)***	1.021 (0.565)	1.021 (0.697)	<i>1.021</i>
<b>0.30</b>	1.004 (0.000)***	1.026 (0.000)***	1.035 (0.000)***	1.042 (0.000)***	1.046 (0.000)***	1.047 (0.000)***	1.048 (0.017)**	1.048 (0.930)	<i>1.048</i>
<b>0.40</b>	1.024 (0.000)***	1.057 (0.000)***	1.071 (0.000)***	1.083 (0.000)***	1.088 (0.000)***	1.089 (0.000)***	1.091 (0.014)**	1.091 (0.149)	<i>1.091</i>
<b>0.50</b>	1.064 (0.000)***	1.104 (0.000)***	1.128 (0.000)***	1.142 (0.000)***	1.149 (0.000)***	1.152 (0.000)***	1.154 (0.000)***	1.154 (0.002)***	<i>1.155</i>
<b>0.60</b>	1.108 (0.000)***	1.171 (0.000)***	1.209 (0.000)***	1.227 (0.000)***	1.241 (0.000)***	1.246 (0.000)***	1.249 (0.000)***	1.250 (0.007)***	<i>1.250</i>
<b>0.70</b>	1.178 (0.000)***	1.272 (0.000)***	1.333 (0.000)***	1.364 (0.000)***	1.386 (0.000)***	1.393 (0.000)***	1.399 (0.000)***	1.399 (0.000)***	<i>1.400</i>
<b>0.80</b>	1.275 (0.000)***	1.434 (0.000)***	1.532 (0.000)***	1.598 (0.000)***	1.638 (0.000)***	1.653 (0.000)***	1.663 (0.000)***	1.665 (0.000)***	<i>1.667</i>
<b>0.90</b>	1.425 (0.000)***	1.699 (0.000)***	1.936 (0.000)***	2.095 (0.000)***	2.207 (0.000)***	2.248 (0.000)***	2.285 (0.000)***	2.289 (0.000)***	<i>2.294</i>
<b>1.00</b>	1.652 (0.000)***	2.294 (0.000)***	3.209 (0.000)***	4.510 (0.000)***	7.087 (0.000)***	10.099 (0.000)***	22.361 (0.000)***	31.599 (0.000)***	$\sigma_u$
	<i>5.000</i>	<i>7.071</i>	<i>10.000</i>	<i>14.142</i>	<i>22.361</i>	<i>31.623</i>	<i>70.711</i>	<i>100.000</i>	← ( $\rho = 1$ )

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $s$  for  $\phi = 0.0$  are reported

P-values of t-tests for the averages of  $s$  equalling their population values ( $\sigma_u$ ) are given in parentheses. The population values of

each  $\rho$  (when  $\rho < 1$ ) are given in italics in the far right column and are calculated with  $\sigma_u = +\sqrt{\frac{1}{1-\rho^2}}$ . The population values for

each value of T when  $\rho = 1$  are given in italics in the bottom row and are calculated with  $\sigma_u = +\sqrt{T}$ .

\*, \*\* and \*\*\* denote the averages of  $s$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 24b: Averages of  $s$  in the three-variable LRM ( $\phi = 0.5$ )**

$\rho \downarrow$	← Sample size (T) →								$\sigma_u \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	0.988 (0.000)***	0.993 (0.000)***	0.997 (0.000)***	0.999 (0.009)***	0.999 (0.013)**	1.000 (0.567)	1.000 (0.518)	1.000 (0.595)	<i>1.000</i>
<b>0.10</b>	0.983 (0.000)***	0.993 (0.000)***	1.000 (0.000)***	1.003 (0.000)***	1.004 (0.004)***	1.005 (0.060)*	1.005 (0.133)	1.005 (0.025)**	<i>1.005</i>
<b>0.20</b>	0.988 (0.000)***	1.002 (0.000)***	1.012 (0.000)***	1.016 (0.000)***	1.019 (0.000)***	1.020 (0.000)***	1.020 (0.008)***	1.021 (0.380)	<i>1.021</i>
<b>0.30</b>	0.999 (0.000)***	1.021 (0.000)***	1.035 (0.000)***	1.041 (0.000)***	1.045 (0.000)***	1.046 (0.000)***	1.048 (0.000)***	1.048 (0.001)***	<i>1.048</i>
<b>0.40</b>	1.020 (0.000)***	1.052 (0.000)***	1.070 (0.000)***	1.081 (0.000)***	1.087 (0.000)***	1.089 (0.000)***	1.091 (0.000)***	1.091 (0.140)	<i>1.091</i>
<b>0.50</b>	1.048 (0.000)***	1.097 (0.000)***	1.125 (0.000)***	1.139 (0.000)***	1.148 (0.000)***	1.151 (0.000)***	1.154 (0.000)***	1.155 (0.059)*	<i>1.155</i>
<b>0.60</b>	1.095 (0.000)***	1.161 (0.000)***	1.201 (0.000)***	1.223 (0.000)***	1.240 (0.000)***	1.245 (0.000)***	1.249 (0.000)***	1.249 (0.000)***	<i>1.250</i>
<b>0.70</b>	1.153 (0.000)***	1.263 (0.000)***	1.327 (0.000)***	1.362 (0.000)***	1.385 (0.000)***	1.392 (0.000)***	1.398 (0.000)***	1.399 (0.000)***	<i>1.400</i>
<b>0.80</b>	1.247 (0.000)***	1.414 (0.000)***	1.525 (0.000)***	1.591 (0.000)***	1.636 (0.000)***	1.651 (0.000)***	1.663 (0.000)***	1.665 (0.000)***	<i>1.667</i>
<b>0.90</b>	1.380 (0.000)***	1.679 (0.000)***	1.925 (0.000)***	2.080 (0.000)***	2.201 (0.000)***	2.247 (0.000)***	2.283 (0.000)***	2.289 (0.000)***	<i>2.294</i>
<b>1.00</b>	1.585 (0.000)***	2.258 (0.000)***	3.177 (0.000)***	4.481 (0.000)***	7.082 (0.000)***	9.991 (0.000)***	22.414 (0.000)***	31.728 (0.000)***	$\sigma_u$
	<i>5.000</i>	<i>7.071</i>	<i>10.000</i>	<i>14.142</i>	<i>22.361</i>	<i>31.623</i>	<i>70.711</i>	<i>100.000</i>	← ( $\rho = 1$ )

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_{0,0} = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $s$  for  $\phi = 0.5$  are reported

P-values of t-tests for the averages of  $s$  equalling their population values ( $\sigma_u$ ) are given in parentheses. The population values of

each  $\rho$  (when  $\rho < 1$ ) are given in italics in the far right column and are calculated with  $\sigma_u = +\sqrt{\frac{1}{1-\rho^2}}$ . The population values for

each value of T when  $\rho = 1$  are given in italics in the bottom row and are calculated with  $\sigma_u = +\sqrt{T}$ .

\*, \*\* and \*\*\* denote the averages of  $s$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 24c: Averages of  $s$  in the three-variable LRM ( $\phi = 0.9$ )**

$\rho \downarrow$	← Sample size (T) →								$\sigma_u \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	0.987 (0.000)***	0.996 (0.000)***	0.997 (0.000)***	0.998 (0.000)***	0.999 (0.011)**	0.999 (0.020)**	1.000 (0.984)	1.000 (0.810)	<i>1.000</i>
<b>0.10</b>	0.981 (0.000)***	0.993 (0.000)***	0.998 (0.000)***	1.002 (0.000)***	1.004 (0.000)***	1.004 (0.016)**	1.005 (0.002)***	1.005 (0.164)	<i>1.005</i>
<b>0.20</b>	0.986 (0.000)***	1.003 (0.000)***	1.010 (0.000)***	1.016 (0.000)***	1.018 (0.000)***	1.020 (0.000)***	1.020 (0.001)***	1.020 (0.052)*	<i>1.021</i>
<b>0.30</b>	0.990 (0.000)***	1.020 (0.000)***	1.034 (0.000)***	1.040 (0.000)***	1.045 (0.000)***	1.047 (0.000)***	1.048 (0.000)***	1.048 (0.085)*	<i>1.048</i>
<b>0.40</b>	1.007 (0.000)***	1.048 (0.000)***	1.068 (0.000)***	1.079 (0.000)***	1.086 (0.000)***	1.089 (0.000)***	1.091 (0.259)	1.091 (0.012)**	<i>1.091</i>
<b>0.50</b>	1.036 (0.000)***	1.088 (0.000)***	1.117 (0.000)***	1.136 (0.000)***	1.147 (0.000)***	1.151 (0.000)***	1.154 (0.000)***	1.154 (0.002)***	<i>1.155</i>
<b>0.60</b>	1.072 (0.000)***	1.149 (0.000)***	1.196 (0.000)***	1.222 (0.000)***	1.238 (0.000)***	1.245 (0.000)***	1.249 (0.000)***	1.249 (0.000)***	<i>1.250</i>
<b>0.70</b>	1.129 (0.000)***	1.236 (0.000)***	1.311 (0.000)***	1.352 (0.000)***	1.380 (0.000)***	1.392 (0.000)***	1.398 (0.000)***	1.399 (0.000)***	<i>1.400</i>
<b>0.80</b>	1.206 (0.000)***	1.379 (0.000)***	1.499 (0.000)***	1.577 (0.000)***	1.629 (0.000)***	1.646 (0.000)***	1.663 (0.000)***	1.665 (0.000)***	<i>1.667</i>
<b>0.90</b>	1.319 (0.000)***	1.616 (0.000)***	1.868 (0.000)***	2.052 (0.000)***	2.187 (0.000)***	2.239 (0.000)***	2.283 (0.000)***	2.288 (0.000)***	<i>2.294</i>
<b>1.00</b>	1.481 (0.000)***	2.110 (0.000)***	3.044 (0.000)***	4.356 (0.000)***	6.973 (0.000)***	9.817 (0.000)***	22.441 (0.000)***	31.508 (0.000)***	$\sigma_u$
	<i>5.000</i>	<i>7.071</i>	<i>10.000</i>	<i>14.142</i>	<i>22.361</i>	<i>31.623</i>	<i>70.711</i>	<i>100.000</i>	← ( $\rho = 1$ )

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $s$  for  $\phi = 0.9$  are reported

P-values of t-tests for the averages of  $s$  equalling their population values ( $\sigma_u$ ) are given in parentheses. The population values of

each  $\rho$  (when  $\rho < 1$ ) are given in italics in the far right column and are calculated with  $\sigma_u = +\sqrt{\frac{1}{1-\rho^2}}$ . The population values for

each value of T when  $\rho = 1$  are given in italics in the bottom row and are calculated with  $\sigma_u = +\sqrt{T}$ .

\*, \*\* and \*\*\* denote the averages of  $s$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 24d: Averages of  $s$  in the three-variable LRM ( $\phi = 1.0$ )**

$\rho \downarrow$	← Sample size (T) →								$\sigma_u \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	0.987 (0.000)***	0.995 (0.000)***	0.997 (0.000)***	0.999 (0.111)	1.000 (0.769)	1.000 (0.514)	1.000 (0.007)***	1.000 (0.661)	<i>1.000</i>
<b>0.10</b>	0.980 (0.000)***	0.992 (0.000)***	0.999 (0.000)***	1.003 (0.000)***	1.004 (0.000)***	1.005 (0.285)	1.005 (0.776)	1.005 (0.043)**	<i>1.005</i>
<b>0.20</b>	0.983 (0.000)***	1.002 (0.000)***	1.011 (0.000)***	1.016 (0.000)***	1.019 (0.000)***	1.020 (0.247)	1.020 (0.005)***	1.021 (0.490)	<i>1.021</i>
<b>0.30</b>	0.994 (0.000)***	1.018 (0.000)***	1.032 (0.000)***	1.040 (0.000)***	1.045 (0.000)***	1.047 (0.000)***	1.048 (0.001)***	1.048 (0.032)**	<i>1.048</i>
<b>0.40</b>	1.010 (0.000)***	1.044 (0.000)***	1.066 (0.000)***	1.079 (0.000)***	1.086 (0.000)***	1.089 (0.000)***	1.091 (0.000)***	1.091 (0.001)***	<i>1.091</i>
<b>0.50</b>	1.034 (0.000)***	1.086 (0.000)***	1.118 (0.000)***	1.136 (0.000)***	1.147 (0.000)***	1.150 (0.000)***	1.154 (0.000)***	1.154 (0.000)***	<i>1.155</i>
<b>0.60</b>	1.070 (0.000)***	1.147 (0.000)***	1.193 (0.000)***	1.219 (0.000)***	1.238 (0.000)***	1.245 (0.000)***	1.249 (0.000)***	1.249 (0.000)***	<i>1.250</i>
<b>0.70</b>	1.122 (0.000)***	1.228 (0.000)***	1.302 (0.000)***	1.347 (0.000)***	1.379 (0.000)***	1.390 (0.000)***	1.398 (0.000)***	1.400 (0.000)***	<i>1.400</i>
<b>0.80</b>	1.190 (0.000)***	1.363 (0.000)***	1.488 (0.000)***	1.570 (0.000)***	1.625 (0.000)***	1.644 (0.000)***	1.662 (0.000)***	1.665 (0.000)***	<i>1.667</i>
<b>0.90</b>	1.297 (0.000)***	1.579 (0.000)***	1.827 (0.000)***	2.024 (0.000)***	2.167 (0.000)***	2.230 (0.000)***	2.280 (0.000)***	2.287 (0.000)***	<i>2.294</i>
<b>1.00</b>	1.442 (0.000)***	1.975 (0.000)***	2.790 (0.000)***	3.901 (0.000)***	6.148 (0.000)***	8.657 (0.000)***	19.380 (0.000)***	27.233 (0.000)***	$\sigma_u$
	<i>5.000</i>	<i>7.071</i>	<i>10.000</i>	<i>14.142</i>	<i>22.361</i>	<i>31.623</i>	<i>70.711</i>	<i>100.000</i>	← ( $\rho = 1$ )

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_{0,0} = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $s$  for  $\phi = 1.0$  are reported

P-values of t-tests for the averages of  $s$  equalling their population values ( $\sigma_u$ ) are given in parentheses. The population values of each  $\rho$  (when  $\rho < 1$ ) are given in italics in the far right column and are calculated with  $\sigma_u = +\sqrt{\frac{1}{1-\rho^2}}$ . The population values for

each value of T when  $\rho = 1$  are given in italics in the bottom row and are calculated with  $\sigma_u = +\sqrt{T}$ .

\*, \*\* and \*\*\* denote the averages of  $s$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 25a: Averages of  $d$  in the three-variable LRM ( $\phi = 0.0$ )**

$\rho \downarrow$	← Sample size (T) →								$DW \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	2.072 (0.000)***	2.040 (0.000)***	2.017 (0.000)***	2.011 (0.000)***	2.004 (0.000)***	2.002 (0.005)***	2.000 (0.274)	2.000 (0.972)	<i>2.000</i>
<b>0.10</b>	1.909 (0.000)***	1.859 (0.000)***	1.833 (0.000)***	1.817 (0.000)***	1.807 (0.000)***	1.804 (0.000)***	1.801 (0.001)***	1.801 (0.001)***	<i>1.800</i>
<b>0.20</b>	1.761 (0.000)***	1.682 (0.000)***	1.644 (0.000)***	1.622 (0.000)***	1.610 (0.000)***	1.604 (0.000)***	1.601 (0.064)*	1.600 (0.102)	<i>1.600</i>
<b>0.30</b>	1.615 (0.000)***	1.508 (0.000)***	1.456 (0.000)***	1.428 (0.000)***	1.412 (0.000)***	1.406 (0.000)***	1.401 (0.000)***	1.400 (0.049)**	<i>1.400</i>
<b>0.40</b>	1.461 (0.000)***	1.332 (0.000)***	1.271 (0.000)***	1.232 (0.000)***	1.214 (0.000)***	1.206 (0.000)***	1.202 (0.000)***	1.201 (0.000)***	<i>1.200</i>
<b>0.50</b>	1.308 (0.000)***	1.154 (0.000)***	1.081 (0.000)***	1.041 (0.000)***	1.016 (0.000)***	1.008 (0.000)***	1.002 (0.000)***	1.001 (0.000)***	<i>1.000</i>
<b>0.60</b>	1.168 (0.000)***	0.988 (0.000)***	0.893 (0.000)***	0.850 (0.000)***	0.819 (0.000)***	0.809 (0.000)***	0.802 (0.000)***	0.801 (0.000)***	<i>0.800</i>
<b>0.70</b>	1.025 (0.000)***	0.812 (0.000)***	0.705 (0.000)***	0.653 (0.000)***	0.621 (0.000)***	0.610 (0.000)***	0.602 (0.000)***	0.601 (0.000)***	<i>0.600</i>
<b>0.80</b>	0.895 (0.000)***	0.648 (0.000)***	0.524 (0.000)***	0.458 (0.000)***	0.423 (0.000)***	0.411 (0.000)***	0.402 (0.000)***	0.401 (0.000)***	<i>0.400</i>
<b>0.90</b>	0.768 (0.000)***	0.496 (0.000)***	0.342 (0.000)***	0.268 (0.000)***	0.226 (0.000)***	0.213 (0.000)***	0.203 (0.000)***	0.201 (0.000)***	<i>0.200</i>
<b>1.00</b>	0.678 (0.000)***	0.369 (0.000)***	0.193 (0.000)***	0.099 (0.000)***	0.040 (0.000)***	0.020 (0.000)***	0.004 (0.000)***	0.002 (0.000)***	<i>0.000</i>

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $d$  for  $\phi = 0.0$  are reported

P-values of t-tests for the averages of  $d$  equalling their population values ( $DW$ ) are given in parentheses. The population value for each value of  $\rho$  (for all T) are given in italics in the far right column and are calculated with  $DW \cong 2(1 - \rho)$ .

\*, \*\* and \*\*\* denote the averages of  $d$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 25b: Averages of  $d$  in the three-variable LRM ( $\phi = 0.5$ )**

$\rho \downarrow$	← Sample size (T) →								$DW \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	2.100 (0.000)***	2.053 (0.000)***	2.029 (0.000)***	2.017 (0.000)***	2.005 (0.000)***	2.003 (0.000)***	2.001 (0.050)**	2.000 (0.044)**	<i>2.000</i>
<b>0.10</b>	1.949 (0.000)***	1.882 (0.000)***	1.844 (0.000)***	1.820 (0.000)***	1.808 (0.000)***	1.805 (0.000)***	1.801 (0.010)***	1.800 (0.198)	<i>1.800</i>
<b>0.20</b>	1.794 (0.000)***	1.702 (0.000)***	1.656 (0.000)***	1.627 (0.000)***	1.610 (0.000)***	1.605 (0.000)***	1.601 (0.000)***	1.601 (0.001)***	<i>1.600</i>
<b>0.30</b>	1.653 (0.000)***	1.533 (0.000)***	1.462 (0.000)***	1.434 (0.000)***	1.413 (0.000)***	1.407 (0.000)***	1.402 (0.000)***	1.401 (0.000)***	<i>1.400</i>
<b>0.40</b>	1.502 (0.000)***	1.357 (0.000)***	1.281 (0.000)***	1.239 (0.000)***	1.215 (0.000)***	1.209 (0.000)***	1.202 (0.000)***	1.201 (0.000)***	<i>1.200</i>
<b>0.50</b>	1.360 (0.000)***	1.182 (0.000)***	1.090 (0.000)***	1.046 (0.000)***	1.019 (0.000)***	1.010 (0.000)***	1.002 (0.000)***	1.001 (0.000)***	<i>1.000</i>
<b>0.60</b>	1.216 (0.000)***	1.012 (0.000)***	0.905 (0.000)***	0.853 (0.000)***	0.822 (0.000)***	0.810 (0.000)***	0.802 (0.000)***	0.801 (0.000)***	<i>0.800</i>
<b>0.70</b>	1.076 (0.000)***	0.837 (0.000)***	0.717 (0.000)***	0.657 (0.000)***	0.622 (0.000)***	0.612 (0.000)***	0.602 (0.000)***	0.601 (0.000)***	<i>0.600</i>
<b>0.80</b>	0.949 (0.000)***	0.675 (0.000)***	0.535 (0.000)***	0.466 (0.000)***	0.425 (0.000)***	0.413 (0.000)***	0.403 (0.000)***	0.402 (0.000)***	<i>0.400</i>
<b>0.90</b>	0.834 (0.000)***	0.520 (0.000)***	0.355 (0.000)***	0.275 (0.000)***	0.229 (0.000)***	0.214 (0.000)***	0.203 (0.000)***	0.201 (0.000)***	<i>0.200</i>
<b>1.00</b>	0.744 (0.000)***	0.394 (0.000)***	0.206 (0.000)***	0.104 (0.000)***	0.042 (0.000)***	0.021 (0.000)***	0.004 (0.000)***	0.002 (0.000)***	<i>0.000</i>

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_{0,0} = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $d$  for  $\phi = 0.5$  are reported

P-values of t-tests for the averages of  $d$  equalling their population values ( $DW$ ) are given in parentheses. The population value for each value of  $\rho$  (for all T) are given in italics in the far right column and are calculated with  $DW \cong 2(1 - \rho)$ .

\*, \*\* and \*\*\* denote the averages of  $d$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 25c: Averages of  $d$  in the three-variable LRM ( $\phi = 0.9$ )**

$\rho \downarrow$	← Sample size (T) →								$DW \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	2.130 (0.000)***	2.070 (0.000)***	2.037 (0.000)***	2.019 (0.000)***	2.008 (0.000)***	2.003 (0.000)***	2.001 (0.000)***	2.000 (0.050)**	<i>2.000</i>
<b>0.10</b>	1.980 (0.000)***	1.901 (0.000)***	1.848 (0.000)***	1.826 (0.000)***	1.810 (0.000)***	1.804 (0.000)***	1.801 (0.000)***	1.800 (0.104)	<i>1.800</i>
<b>0.20</b>	1.830 (0.000)***	1.716 (0.000)***	1.665 (0.000)***	1.630 (0.000)***	1.613 (0.000)***	1.606 (0.000)***	1.601 (0.000)***	1.601 (0.000)***	<i>1.600</i>
<b>0.30</b>	1.680 (0.000)***	1.541 (0.000)***	1.472 (0.000)***	1.439 (0.000)***	1.414 (0.000)***	1.407 (0.000)***	1.402 (0.000)***	1.401 (0.000)***	<i>1.400</i>
<b>0.40</b>	1.541 (0.000)***	1.376 (0.000)***	1.286 (0.000)***	1.244 (0.000)***	1.219 (0.000)***	1.209 (0.000)***	1.201 (0.000)***	1.201 (0.000)***	<i>1.200</i>
<b>0.50</b>	1.388 (0.000)***	1.201 (0.000)***	1.101 (0.000)***	1.050 (0.000)***	1.021 (0.000)***	1.009 (0.000)***	1.002 (0.000)***	1.001 (0.000)***	<i>1.000</i>
<b>0.60</b>	1.254 (0.000)***	1.031 (0.000)***	0.914 (0.000)***	0.856 (0.000)***	0.822 (0.000)***	0.810 (0.000)***	0.802 (0.000)***	0.801 (0.000)***	<i>0.800</i>
<b>0.70</b>	1.124 (0.000)***	0.866 (0.000)***	0.729 (0.000)***	0.664 (0.000)***	0.625 (0.000)***	0.612 (0.000)***	0.603 (0.000)***	0.601 (0.000)***	<i>0.600</i>
<b>0.80</b>	1.001 (0.000)***	0.702 (0.000)***	0.547 (0.000)***	0.470 (0.000)***	0.427 (0.000)***	0.414 (0.000)***	0.403 (0.000)***	0.401 (0.000)***	<i>0.400</i>
<b>0.90</b>	0.894 (0.000)***	0.555 (0.000)***	0.371 (0.000)***	0.281 (0.000)***	0.231 (0.000)***	0.215 (0.000)***	0.203 (0.000)***	0.202 (0.000)***	<i>0.200</i>
<b>1.00</b>	0.814 (0.000)***	0.444 (0.000)***	0.227 (0.000)***	0.114 (0.000)***	0.045 (0.000)***	0.022 (0.000)***	0.004 (0.000)***	0.002 (0.000)***	<i>0.000</i>

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_0 = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $d$  for  $\phi = 0.9$  are reported

P-values of t-tests for the averages of  $d$  equalling their population values ( $DW$ ) are given in parentheses. The population value for each value of  $\rho$  (for all T) are given in italics in the far right column and are calculated with  $DW \cong 2(1 - \rho)$ .

\*, \*\* and \*\*\* denote the averages of  $d$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 25d: Averages of  $d$  in the three-variable LRM ( $\phi = 1.0$ )**

$\rho \downarrow$	← Sample size (T) →								$DW \downarrow$
	25	50	100	200	500	1000	5000	10000	
<b>0.00</b>	2.132 (0.000)***	2.075 (0.000)***	2.042 (0.000)***	2.020 (0.000)***	2.008 (0.000)***	2.004 (0.000)***	2.001 (0.007)***	2.000 (0.286)	<i>2.000</i>
<b>0.10</b>	1.987 (0.000)***	1.896 (0.000)***	1.850 (0.000)***	1.824 (0.000)***	1.810 (0.000)***	1.805 (0.000)***	1.801 (0.001)***	1.801 (0.003)***	<i>1.800</i>
<b>0.20</b>	1.839 (0.000)***	1.718 (0.000)***	1.662 (0.000)***	1.633 (0.000)***	1.612 (0.000)***	1.606 (0.000)***	1.602 (0.000)***	1.601 (0.000)***	<i>1.600</i>
<b>0.30</b>	1.689 (0.000)***	1.546 (0.000)***	1.478 (0.000)***	1.438 (0.000)***	1.417 (0.000)***	1.407 (0.000)***	1.401 (0.000)***	1.401 (0.000)***	<i>1.400</i>
<b>0.40</b>	1.541 (0.000)***	1.377 (0.000)***	1.288 (0.000)***	1.243 (0.000)***	1.217 (0.000)***	1.209 (0.000)***	1.201 (0.000)***	1.201 (0.000)***	<i>1.200</i>
<b>0.50</b>	1.395 (0.000)***	1.211 (0.000)***	1.100 (0.000)***	1.048 (0.000)***	1.021 (0.000)***	1.010 (0.000)***	1.002 (0.000)***	1.001 (0.000)***	<i>1.000</i>
<b>0.60</b>	1.263 (0.000)***	1.031 (0.000)***	0.915 (0.000)***	0.858 (0.000)***	0.823 (0.000)***	0.811 (0.000)***	0.802 (0.000)***	0.801 (0.000)***	<i>0.800</i>
<b>0.70</b>	1.129 (0.000)***	0.872 (0.000)***	0.733 (0.000)***	0.666 (0.000)***	0.626 (0.000)***	0.612 (0.000)***	0.603 (0.000)***	0.601 (0.000)***	<i>0.600</i>
<b>0.80</b>	1.015 (0.000)***	0.709 (0.000)***	0.549 (0.000)***	0.472 (0.000)***	0.428 (0.000)***	0.414 (0.000)***	0.403 (0.000)***	0.401 (0.000)***	<i>0.400</i>
<b>0.90</b>	0.907 (0.000)***	0.567 (0.000)***	0.380 (0.000)***	0.284 (0.000)***	0.232 (0.000)***	0.215 (0.000)***	0.203 (0.000)***	0.202 (0.000)***	<i>0.200</i>
<b>1.00</b>	0.836 (0.000)***	0.469 (0.000)***	0.244 (0.000)***	0.126 (0.000)***	0.051 (0.000)***	0.026 (0.000)***	0.005 (0.000)***	0.003 (0.000)***	<i>0.000</i>

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t}$ :  $Y_{0t} = \rho Y_{0,t-1} + e_{0t}$ ,  $Y_{0,0} = 0$ ,  $e_{0t} \sim N(0, 1)$

DGP  $X_{1t}$ :  $X_{1t} = \phi X_{1,t-1} + e_{1t}$ ,  $X_{1,0} = 0$ ,  $e_{1t} \sim N(0, 1)$

DGP  $X_{2t}$ :  $X_{2t} = X_{2,t-1} + v_{2t}$ ,  $X_{2,0} = 0$ ,  $v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The averages of  $d$  for  $\phi = 1.0$  are reported

P-values of t-tests for the averages of  $d$  equalling their population values ( $DW$ ) are given in parentheses. The population value for each value of  $\rho$  (for all T) are given in italics in the far right column and are calculated with  $DW \cong 2(1 - \rho)$ .

\*, \*\* and \*\*\* denote the averages of  $d$  are significantly different from their population values at the 10%, 5% and 1% levels, respectively

**Table 26a: Empirical sizes of  $d$  (for  $H_0 : d = 2$ ) in the three-variable LRM ( $\phi = 0.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	1.04%	1.74%	2.46%	3.07%	0.24%	0.00%	0.00%	0.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.10</b>	3.07%	7.54%	15.43%	31.63%	25.38%	18.52%	2.93%	0.21%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	6.88%	21.71%	49.21%	81.91%	94.24%	98.94%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	14.89%	44.01%	82.32%	98.91%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	26.59%	70.21%	97.02%	99.98%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	42.18%	87.89%	99.75%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	57.57%	95.90%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	71.63%	99.01%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	81.56%	99.71%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	88.45%	99.95%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	91.70%	99.98%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

DGP  $Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$

DGP  $X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$

DGP  $X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$

Regression:  $Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$

The empirical sizes (in percentages) of  $d$  (using a 5% nominal size) for  $\phi = 0.0$  are reported. The lower bounds of the DW-distribution are used for critical values.

P-values of t-tests for the empirical size of  $d$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $d$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 26b: Empirical sizes of  $d$  (for  $H_0 : d = 2$ ) in the three-variable LRM ( $\phi = 0.5$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.85%	1.54%	2.37%	2.60%	0.23%	0.00%	0.00%	0.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.10</b>	2.75%	6.20%	14.38%	30.55%	24.72%	18.16%	3.11%	0.37%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	5.77%	19.77%	46.27%	80.74%	94.11%	98.90%	100.00%	100.00%
	(0.001)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	12.08%	41.27%	82.00%	98.47%	99.99%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	23.29%	66.42%	96.50%	99.99%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	36.70%	85.34%	99.54%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	51.96%	94.75%	99.98%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	66.23%	98.54%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	76.96%	99.61%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	84.72%	99.94%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	88.26%	99.96%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

$$\text{DGP } Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$$

$$\text{DGP } X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$$

$$\text{DGP } X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$$

$$\text{Regression: } Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$$

The empirical sizes (in percentages) of  $d$  (using a 5% nominal size) for  $\phi = 0.5$  are reported. The lower bounds of the DW-distribution are used for critical values.

P-values of t-tests for the empirical size of  $d$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $d$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 26c: Empirical sizes of  $d$  (for  $H_0 : d = 2$ ) in the three-variable LRM ( $\phi = 0.9$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.73%	1.57%	1.88%	2.72%	0.17%	0.00%	0.00%	0.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.10</b>	2.07%	5.62%	13.58%	29.00%	24.40%	18.89%	3.01%	0.44%
	(0.000)***	(0.007)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	5.02%	18.11%	44.52%	80.32%	94.15%	98.83%	100.00%	100.00%
	(0.927)	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	10.70%	39.99%	80.15%	98.82%	99.98%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	20.48%	63.86%	96.14%	99.97%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	34.15%	83.73%	99.55%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	48.44%	94.42%	99.98%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	61.85%	98.22%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	72.15%	99.34%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	79.89%	99.91%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	84.34%	99.86%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

$$\text{DGP } Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_0 = 0, \quad e_{0t} \sim N(0, 1)$$

$$\text{DGP } X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$$

$$\text{DGP } X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$$

$$\text{Regression: } Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$$

The empirical sizes (in percentages) of  $d$  (using a 5% nominal size) for  $\phi = 0.9$  are reported. The lower bounds of the DW-distribution are used for critical values.

P-values of t-tests for the empirical size of  $d$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $d$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively

**Table 26d: Empirical sizes of  $d$  (for  $H_0 : d = 2$ ) in the three-variable LRM ( $\phi = 1.0$ )**

$\rho \downarrow$	← Sample size (T) →							
	25	50	100	200	500	1000	5000	10000
<b>0.00</b>	0.57%	1.39%	1.85%	2.44%	0.17%	0.01%	0.00%	0.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.10</b>	1.99%	5.89%	13.81%	29.72%	24.36%	18.21%	3.08%	0.36%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.20</b>	4.76%	17.89%	44.66%	80.11%	94.05%	98.86%	100.00%	100.00%
	(0.260)	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.30</b>	10.68%	39.16%	79.33%	98.38%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.40</b>	19.55%	63.98%	96.09%	99.99%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.50</b>	33.12%	82.72%	99.56%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.60</b>	47.75%	94.11%	99.97%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.70</b>	61.50%	97.76%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.80</b>	71.68%	99.59%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>0.90</b>	78.97%	99.80%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
<b>1.00</b>	83.65%	99.92%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Number of samples (replications) = 10000

Maximum observations in a sample = 10500, Number of discarded initial observations = 500

$$\text{DGP } Y_{0t} : Y_{0t} = \rho Y_{0,t-1} + e_{0t}, \quad Y_{0,0} = 0, \quad e_{0t} \sim N(0, 1)$$

$$\text{DGP } X_{1t} : X_{1t} = \phi X_{1,t-1} + e_{1t}, \quad X_{1,0} = 0, \quad e_{1t} \sim N(0, 1)$$

$$\text{DGP } X_{2t} : X_{2t} = X_{2,t-1} + v_{2t}, \quad X_{2,0} = 0, \quad v_{2t} \sim N(0, 1)$$

$$\text{Regression: } Y_{0t} = \hat{\alpha} + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \hat{u}_t$$

The empirical sizes (in percentages) of  $d$  (using a 5% nominal size) for  $\phi = 1.0$  are reported. The lower bounds of the DW-distribution are used for critical values.

P-values of t-tests for the empirical size of  $d$  equalling 5% are given in parentheses

\*, \*\* and \*\*\* denote the empirical sizes of  $d$  are significantly different from 5% at the 10%, 5% and 1% levels, respectively